DUNMAN SECONDARY SCHOOL

CANDIDATE NAME			
CLASS		INDEX NUMBER	

PRELIMINARY EXAMINATION 2025
SECONDARY 4 EXPRESS

ADDITIONAL MATHEMATICS **4049/01**

Paper 1 20 August 2025

2 hours 15 minutes

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Calculator Model

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total marks for this paper is 90.

Total Marks

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formula for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 The line $y = x - 4$ and the curve $y = \dfrac{2}{x-3}$ intersect at the points A and B.

Find the length of AB. [6]

2 **(a)** State the range of $\tan^{-1} p$. [1]

(b) Given that $\cos^{-1} k = x$, where $0 \le x \le \frac{\pi}{2}$, find the principal value of $\cos^{-1}(-k)$. [1]

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3 Explain why $3x^2 + 15x + 20$ cannot be smaller than 1. [3]

4 **(a)** Express $\dfrac{x^2+3x+2}{x^2(2x+1)}$ in partial fractions. [5]

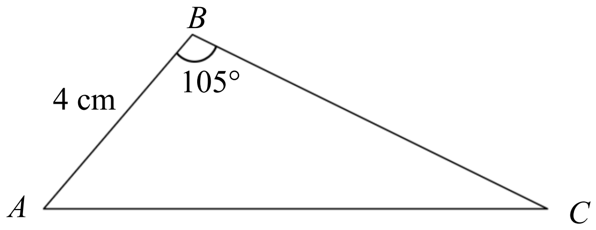
(b) Hence integrate $\dfrac{x^2+3x+2}{x^2(2x+1)}$ with respect to x. [3]

5 Solve the equation $3\log_5 y - \log_y 5 = 2$. [4]

6 **(a)** Show that $\sin 105° = \dfrac{\sqrt{6}+\sqrt{2}}{4}$. [3]

(b) The diagram below shows triangle ABC, such that $AB = 4$ cm, angle $ABC = 105°$ and the area of triangle ABC is $10 + 2\sqrt{3}$ cm^2.

Find BC in the form $\sqrt{2}\left(a\sqrt{3}+b\right)$, where a and b are integers. [3]



7 **(a)** Given that $u = \left(\frac{3}{2}\right)^x$, express $9^x + 6^x = 6\left(4^x\right)$ as a quadratic equation in u. [3]

(b) Hence, or otherwise, solve $9^x + 6^x = 6\left(4^x\right)$. [3]

8 The expression $2x^3 + ax^2 + bx + 10$, where a and b are constants, has a factor of $x + 2$ and leaves a remainder of 10 when divided by $x - 3$.
Find the values of a and of b. [4]

9 A cyclist starts from rest from a point A and travels in a straight line until she comes to a rest at a point B. During the motion, her velocity, v m/s, is given by $v = 12t - \frac{3}{2}t^2$, where t is the time in seconds after leaving A.

(a) Find the time taken for the cyclist to travel from A to B. [1]

(b) Find the distance AB. [3]

(c) Find the acceleration of the cyclist when $t = 5$. [2]

10 The equation of a curve is $y = xe^{-3x}$.

Find the stationary point(s) and determine the nature of the stationary point(s). [6]

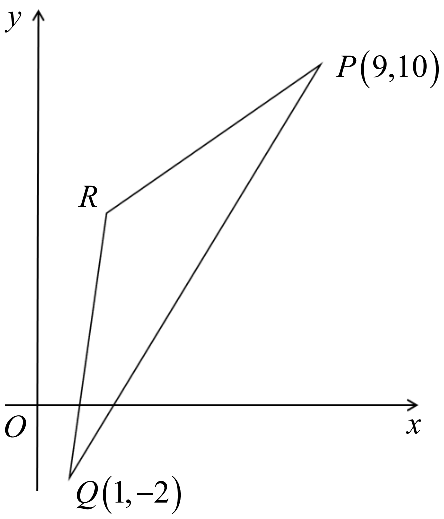
11 **(a)** Sketch the graphs of $y = -2\cos 3x + 1$ and $y = -\frac{2}{\pi}x + 1$ on the same axes, for $0 \le x \le \pi$.

[5]

(b) Explain how the number of solutions for $2\cos 3x = \frac{2}{\pi}x$ can be found using the graphs.

[1]

12



The diagram shows a triangle *PQR* with vertices $P(9,10)$ and $Q(1,-2)$ .The point *R* lies on the perpendicular bisector of *PQ* and the equation of the line *QR* is $y = 8x - 10$.

(a) Find the equation of the perpendicular bisector of *PQ*. [4]

(b) Find the coordinates of *R*. [2]

(c) Find the area of triangle *PQR*. [2]

13 The mass, m mg, of a radioactive substance decreases with time, t hours.
It is known that m and t are related by the equation $m = Me^{-kt}$, where M and k are constants.
The table below shows measured values of m and t.

t (hours)	1	2	3	4	5
m (mg)	6.55	5.36	4.40	3.60	2.94

(a) Plot ln m against t and draw a straight line graph. [2]

Use your graph to estimate

(b) the initial mass of the substance, [2]

(c) the value of k. [2]

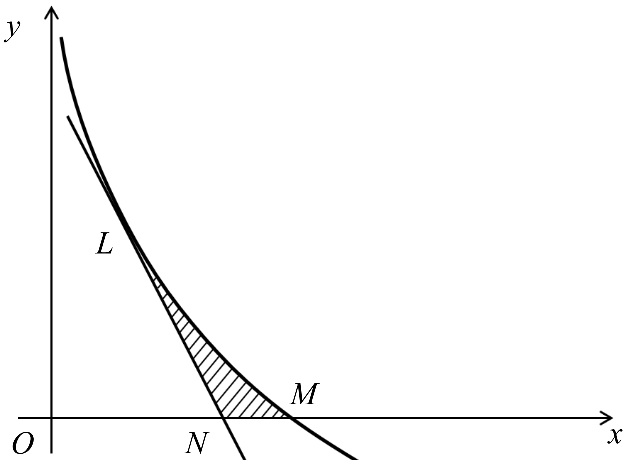
14 A curve is such that $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = -4\sin(2x+\pi)$ and the point $P\left(\frac{\pi}{2}, \pi\right)$ lies on the curve.

The gradient of the curve at P is 3.

(a) Find the equation of the curve. [5]

(b) Justify whether y is increasing when $x = 3$. [2]

15



The diagram shows part of the curve $y = \dfrac{16}{(x+1)^2} - 1$, cutting the x-axis at M.

The tangent at the point L on the curve cuts the x-axis at N.
The gradient of this tangent is -4.
Calculate the area of the shaded region LMN. [12]

Continuation of working space for Question 15.

END OF PAPER

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DUNMAN SECONDARY SCHOOL

CANDIDATE NAME

CLASS

INDEX NUMBER

PRELIMINARY EXAMINATION 2025
SECONDARY 4 EXPRESS / 5 NORMAL ACADEMIC

MATHEMATICS **4052/01**

Paper 1

20 August 2025

2 hours 15 minutes

Solutions

(Updated 1 Dec 2022 by HOD/Math aligned to Marking Scheme for Specimen Papers 4049)

Question	Answer
1	$x-4=\dfrac{2}{x-3}$
	$(x-4)(x-3)=2$
	$x^2-7x+12=2$
	$x^2-7x+10=0$
	$(x-2)(x-5)=0$
	$x=2$ or 5
	When $x = 2$, $y = -2$
	When $x = 5$, $y = 1$
	Length = $\sqrt{(2-5)^2+(-2-1)^2}$ = 4.24 unit
2a	$-90° < \tan^{-1} p < 90°$ or $-\dfrac{\pi}{2} < \tan^{-1} p < \dfrac{\pi}{2}$
2b	$\pi - x$
3	$3x^2+15x+20 = 3(x^2+5x)+20$
	$= 3\left[(x+2.5)^2-(2.5)^2\right]+20$
	$= 3(x+2.5)^2+1.25$
	Since $3(x+2.5)^2+1.25 \geq 1.25$, $3x^2+15x+20$ cannot be smaller than 1.

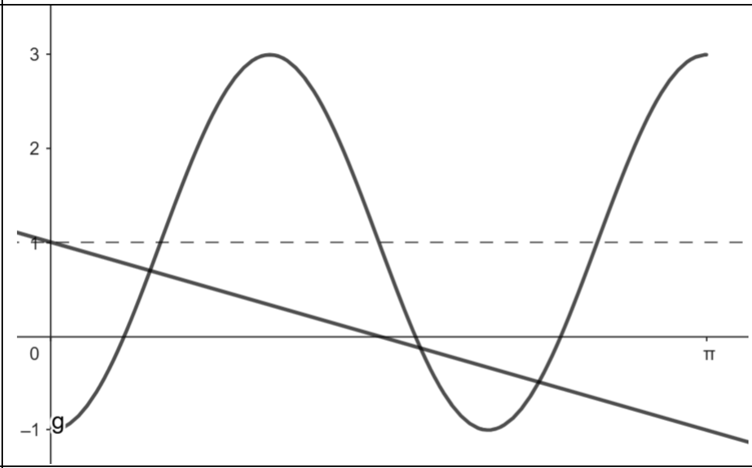
Question	Answer
4a	$\dfrac{x^2+3x+2}{x^2(2x+1)} = \dfrac{A}{x}+\dfrac{B}{x^2}+\dfrac{C}{2x+1}$
	$x^2+3x+2 = Ax(2x+1)+B(2x+1)+Cx^2$
	When $x=-\dfrac{1}{2}$, $\dfrac{1}{4}-\dfrac{3}{2}+2=C\left(\dfrac{1}{4}\right)$ $C=3$
	When $x = 0$, $B = 2$
	Comparing coefficients of x^2, $A = -1$
	$\dfrac{x^2+3x+2}{x^2(2x+1)} = -\dfrac{1}{x}+\dfrac{2}{x^2}+\dfrac{3}{2x+1}$
4b	$\displaystyle\int\dfrac{x^2+3x+2}{x^2(2x+1)}\,\mathrm{d}x=\int-\dfrac{1}{x}+\dfrac{2}{x^2}+\dfrac{3}{2x+1}\,\mathrm{d}x$ $= -\ln x-\dfrac{2}{x}+\dfrac{3}{2}\ln(2x+1)+c$
5	$3\log_5 y-\log_y 5=2$
	$3\log_5 y-\dfrac{\log_5 5}{\log_5 y}=2$
	$3\log_5 y-\dfrac{1}{\log_5 y}=2$
	$3(\log_5 y)^2-2\log_5 y-1=0$
	$(3\log_5 y+1)(\log_5 y-1)=0$
	$\log_5 y=1$ or $-\dfrac{1}{3}$
	$y=5$ or $y=5^{-\frac{1}{3}}=0.585$ (3s.f.)
Question	**Answer**
6a	$\sin(105°)=\sin(60°+45°)$
	$= \sin 60°\cos 45°+\cos 60°\sin 45°$
	$= \dfrac{\sqrt{3}}{2}\cdot\dfrac{1}{\sqrt{2}}+\dfrac{1}{2}\cdot\dfrac{1}{\sqrt{2}}$
	$= \dfrac{\sqrt{3}+1}{2\sqrt{2}}\times\dfrac{\sqrt{2}}{\sqrt{2}}$
	$= \dfrac{\sqrt{6}+\sqrt{2}}{4}$
6b	$\dfrac{1}{2}(4)(BC)\sin 105°=10+2\sqrt{3}$

	$\frac{1}{2}(4)(BC)\left(\frac{\sqrt{6}+\sqrt{2}}{4}\right)=10+2\sqrt{3}$
	$BC=\frac{2\left(10+2\sqrt{3}\right)}{\sqrt{6}+\sqrt{2}}\times\frac{\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}$
	$=\frac{4\left(5+\sqrt{3}\right)\left(\sqrt{6}-\sqrt{2}\right)}{6-2}$
	$=5\sqrt{6}+\sqrt{18}-5\sqrt{2}-\sqrt{6}$
	$=4\sqrt{6}+3\sqrt{2}-5\sqrt{2}$
	$=4\sqrt{6}-2\sqrt{2}$
	$=\sqrt{2}\left(4\sqrt{3}-2\right)$

Question	Answer
7a	$9^x + 6^x = 6\left(4^x\right)$
	$\frac{9^x}{4^x} + \frac{6^x}{4^x} = 6$
	$\frac{3^{2x}}{2^{2x}} + \left(\frac{6}{4}\right)^x - 6 = 0$
	$\left(\frac{3}{2}\right)^{2x} + \left(\frac{3}{2}\right)^x - 6 = 0$
	When $u = \left(\frac{3}{2}\right)^x$, $u^2 + u - 6 = 0$
7b	$(u+3)(u-2) = 0$
	$u = 2$ or -3 (n.a.)
	$\left(\frac{3}{2}\right)^x = 2$
	$x = \frac{\ln 2}{\ln \frac{3}{2}}$ or $\frac{\lg 2}{\lg \frac{3}{2}}$ $= 1.71$
8	When $x = -2$, $2(-2)^3 + a(-2)^2 + b(-2) + 10 = 0$
	$4a - 2b = 6$
	$2a - b = 3$ - eqn 1
	When $x = 3$, $2(3)^3 + a(3)^2 + b(3) + 10 = 10$
	$9a + 3b = -54$
	$3a + b = -18$ - eqn 2
	eqn 1 + eqn 2: $5a = -15$
	$a = -3$
	$\therefore b = -9$
Question	**Answer**
9a	$12t - \frac{3}{2}t^2 = 0$
	$t\left(12 - \frac{3}{2}t\right) = 0$
	$t = 0$ or 8
	$\therefore 8$ s
9b	distance $= \int_0^8 12t - \frac{3}{2}t^2 \, \mathrm{d}x$
	$= \left[6t^2 - \frac{1}{2}t^3\right]_0^8$

	$=[128]-[0]$
	$= 128$ m
9c	$a=12-3t$
	When $t = 5$, $a = -3$ m/s^2

Question	Answer
10	$y = xe^{-3x}$
	$\frac{\mathrm{d}y}{\mathrm{d}x} = e^{-3x}(1) + x\left(-3e^{-3x}\right)$
	$= e^{-3x}(1-3x)$
	$e^{-3x}(1-3x) = 0$
	$x = \frac{1}{3}$
	$y = \frac{1}{3e}$ or 0.123
	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = (1-3x)\left(-3e^{-3x}\right) + e^{-3x}(-3)$
	When $x = \frac{1}{3}$, $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} < 0$
	$\left(\frac{1}{3}, -\frac{1}{3e}\right)$ is a maximum point.

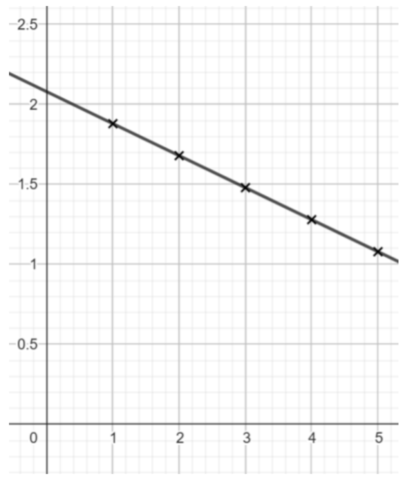
Question	**Answer**
11a	
11b	The $2\cos 3x = \frac{2}{\pi}x$ is equivalent to $-2\cos 3x + 1 = -\frac{2}{\pi}x + 1$ and so its solutions can be found from the interception points of the two graphs.

Question	Answer
12a	Midpoint of $PQ = \left(\frac{1+9}{2}, \frac{-2+10}{2}\right) = (5,4)$
	Gradient of $PQ = \frac{-2-10}{1-9} = \frac{3}{2}$
	Gradient of perpendicular bisector = $-\frac{2}{3}$
	$y-4=-\frac{2}{3}(x-5)$
	$y=-\frac{2}{3}x+\frac{22}{3}$
12b	$-\frac{2}{3}x+\frac{22}{3}=8x-10$
	$-\frac{26}{3}x=-\frac{52}{3}$
	$x=2$
	$R(2,6)$
12c	Area = $\frac{1}{2}\begin{vmatrix} 9 & 2 & 1 & 9 \\ 10 & 6 & -2 & 10 \end{vmatrix}$
	$=\frac{1}{2}\left[(54-4+10)-(20+6-18)\right]$
	= 26 unit^2

Question	Answer
13a	(table and graph below)
13b	$\ln M = 2.08$
	$M = 8.00$
13c	Gradient = -0.2
	$k = 0.2$

13a

t	1	2	3	4	5
m	6.55	5.36	4.40	3.60	2.94
$\ln m$	1.88	1.68	1.48	1.28	1.08



Question	Answer
14a	$\frac{d^2y}{dx^2} = -4\sin(2x+\pi)$
	$\frac{dy}{dx} = 2\cos(2x+\pi) + c_1$
	$3 = 2\cos\left[2\left(\frac{\pi}{2}\right)+\pi\right] + c_1$
	$3 = 2(1) + c_1$
	$c_1 = 1$
	$\frac{dy}{dx} = 2\cos(2x+\pi) + 1$
	$y = \sin(2x+\pi) + x + c_2$
	$\pi = \sin\left[2\left(\frac{\pi}{2}\right)+\pi\right] + \frac{\pi}{2} + c_2$
	$c_2 = \frac{\pi}{2}$
	$y = \sin(2x+\pi) + x + \frac{\pi}{2}$
14b	When $x = 3$, $\frac{dy}{dx} = 2\cos\left[2(3)+\pi\right] + 1 = -0.920$
	y is decreasing when $x = 3$.

Question	Answer
15	$y=\dfrac{16}{(x+1)^2}-1=16(x+1)^{-2}-1$
	$\dfrac{\mathrm{d}y}{\mathrm{d}x}=-32(x+1)^{-3}$
	$-4=-32(x+1)^{-3}$
	$(x+1)^3=8$
	$x=1$
	At L, $y=\dfrac{16}{(1+1)^2}-1=3$
	$y-3=-4(x-1)$
	$y=-4x+7$
	At N, $0=-4x+7$
	$x=1.75$
	At M, $0=\dfrac{16}{(x+1)^2}-1$
	$x=3$ or -5 (n.a.)
	Area $=\left[\int_1^3 16(x+1)^{-2}-1\ \mathrm{d}x\right]-\int_1^{1.75}-4x+7\ \mathrm{d}x$
	$=\left[-16(x+1)^{-1}-x\right]_1^3-\left[-2x^2+7x\right]_1^{1.75}$
	$=2-1.125$
	$=0.875$ unit^2

DUNMAN SECONDARY SCHOOL

CANDIDATE NAME

CLASS

INDEX NUMBER

PRELIMINARY EXAMINATION 2025
SECONDARY 4 EXPRESS

ADDITIONAL MATHEMATICS 4049/02

Paper 2

25 August 2025

2 hours 15 minutes

Candidates answer on the Question Paper.

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Calculator Model

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Total Marks

1. ALGEBRA

Quadratic Equation

For the equation $ax^2+bx+c=0$,

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Binomial expansion

$$(a+b)^n=a^n+\binom{n}{1}a^{n-1}b+\binom{n}{2}a^{n-2}b^2+\ldots+\binom{n}{r}a^{n-r}b^r+\ldots+b^n,$$

where n is a positive integer and $\binom{n}{r}=\frac{n!}{r!(n-r)!}=\frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A+\cos^2 A=1$$

$$\sec^2 A=1+\tan^2 A$$

$$\operatorname{cosec}^2 A=1+\cot^2 A$$

$$\sin(A\pm B)=\sin A\cos B\pm\cos A\sin B$$

$$\cos(A\pm B)=\cos A\cos B\mp\sin A\sin B$$

$$\tan(A\pm B)=\frac{\tan A\pm\tan B}{1\mp\tan A\tan B}$$

$$\sin 2A=2\sin A\cos A$$

$$\cos 2A=\cos^2 A-\sin^2 A=2\cos^2 A-1=1-2\sin^2 A$$

$$\tan 2A=\frac{2\tan A}{1-\tan^2 A}$$

Formula for ΔABC

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$$

$$a^2=b^2+c^2-2bc\cos A$$

$$\Delta=\frac{1}{2}bc\sin A$$

1 **(a)** A ball is thrown vertically upwards. Its height h m, above the ground at time t seconds after being thrown is given by the formula $h = 7 + 20t - 5t^2$.
Find the maximum height attained by the ball and the time at which this occurs. [4]

(b) Find the set of values of the constant k for which the curve $y = x^2 + 6x + k$ lies entirely above the line $y = k(x-1)$. [4]

2 A and B are acute angles, such that $\cos A = \frac{3}{5}$ and $\tan B = \frac{12}{5}$.

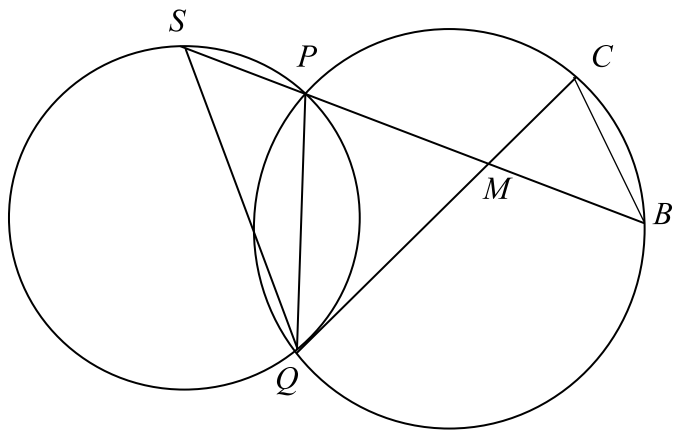
Find the exact values of

(a) $\tan 2A$, [2]

(b) $\sin\left(B + \frac{3\pi}{2}\right)$, [3]

(c) $\cos\frac{B}{2}$. [3]

3 In the diagram, *PQS* lies on a circle and *PCBQ* lies on another circle.
PQ is a common chord of the two circles. *BPS* is a straight line.
QC is a tangent to the circle *PQS* at *Q*. *QC* and *BP* intersect at *M*.



(a) Prove that *BC* is parallel to *QS*. [3]

(b) Prove that triangle *PQM* is similar to triangle *QSM*. [2]

(c) Hence show that $QM^2 = PM \times SM$. [2]

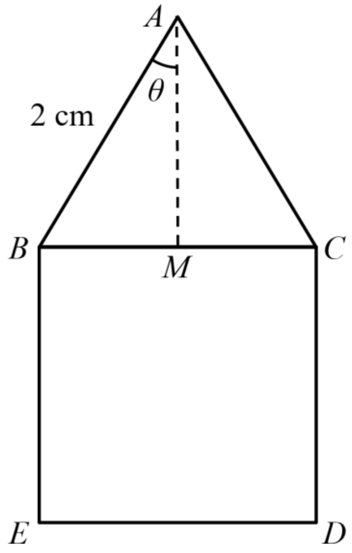
4 **(a)** Show that $\frac{\mathrm{d}}{\mathrm{d}x}(\cot x) = -\operatorname{cosec}^2 x$. [4]

(b) Hence find the value of $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} -\cot^2 x \, \mathrm{d}x$. [4]

5 **(a)** Given that the constant term in the binomial expansion of $\left(x-\frac{k}{x^3}\right)^8$ is 7, find the value of the positive constant *k*. [4]

(b) Using the value of k found in part **(a)**, show that there is no constant term in the expansion of $\left(1+x^4\right)\left(x-\frac{k}{x^3}\right)^8$. [4]

6 The diagram below shows an isosceles triangle ABC, where $AB = AC = 2$ cm, and a square $BCDE$.
M is the midpoint of BC.
Angle $BAM = \theta$ and can vary.



(a) Show that the total area of $ABEDC$ is $16\sin^2\theta + 4\sin\theta\cos\theta$ cm^2. [3]

(b) Show that $16\sin^2\theta + 4\sin\theta\cos\theta = 2\sin 2\theta - 8\cos 2\theta + 8$. [2]

(c) Express $2\sin 2\theta - 8\cos 2\theta$ in the form $R\sin(2\theta - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. [3]

7 **(a)** Prove the identity $\dfrac{\cos^2 x - \sin^2 x}{1 + 2\sin x \cos x} = \dfrac{1 - \tan x}{1 + \tan x}$. [4]

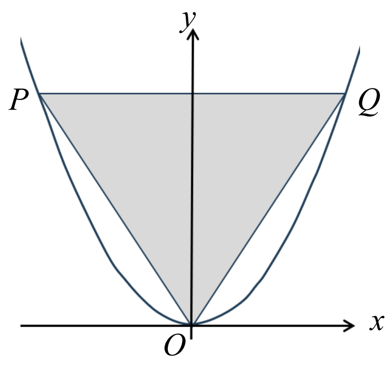
(b) Solve the equation $\dfrac{\cos^2 x - \sin^2 x}{1 + 2\sin x \cos x} = \dfrac{2}{3}\tan x$ for $0 \le x \le 2\pi$. [4]

8 **(a)** Using long division, divide $2x^3 - 7x^2 + 2x + 3$ by $2x - 4$. [2]

(b) Solve $2x^3 - 7x^2 + 2x + 3 = 0$. [4]

(c) Kun Ye claims that the solutions to the equation $2x^3 - 7x^2 + 2x + 3 = 0$ can be used to solve $3y^6 + 2y^4 - 7y^2 + 2 = 0$. Explain how he is correct. [2]

9 The diagram shows part of the curve $y = 3x^2$.
The points P and Q lie on the curve, and have the same y-coordinates.



(a) Show that the area of the triangle OPQ, A cm^2, is given by $A = 3x^3$. [2]

The points P and Q move along the curve such that their y-coordinates are changing at a rate of 3 units per second.

Find, at the point where $x = 4$,

(b) the rate of change of PQ, [4]

(c) the rate of change of A. [3]

10 The lines $y = 9$, $y = -1$ and $3y = 4x + 9$ are tangents to a circle.

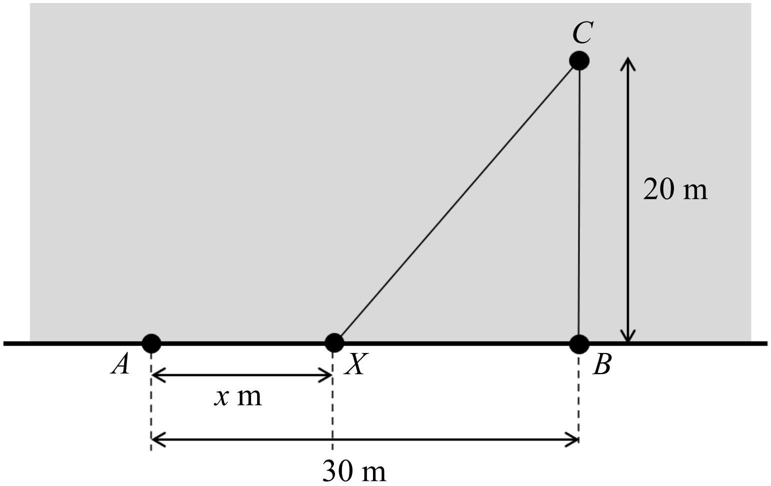
The x-coordinate of the centre of the circle is positive.

(a) Explain why the y-coordinate of the centre of the circle is 4. [1]

(b) By considering the discriminant, or otherwise, show that the x-coordinate of the centre of the circle is 7. [7]

(c) Determine if the point $(3, 0)$ lies within the circle. [2]

11 The diagram shows a lake (shaded) bordered by a straight level road *AXB*.
Point *C* is on the surface of the lake and *B* is the point closest to *C*.
The distance *AB* is 30 m and *BC* is 20 m.



Natalie is at point *A* on the road and wants to get to point *C* as quickly as possible. She can run at a speed of 4 m/s and swim at a speed of 2 m/s.
She realizes that in order to minimize the total time to get to *C*, it is necessary to leave the road at some point *X* between *A* and *B*, and then swim directly to *C*.

Let *AX* be *x* m and *T* be the total time, in seconds, taken by Natalie to get from *A* to *X* and then from *X* to *C*.

(a) Show that $T = \frac{x}{4} + \frac{1}{2}\sqrt{x^2 - 60x + 1300}$. [2]

(b) Find an expression for $\frac{\mathrm{d}T}{\mathrm{d}x}$. [2]

(c) Find the value of x for which the total time taken is minimum. You are **not** required to justify that this value of x leads to the minimum total time. [4]

END OF PAPER

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DUNMAN SECONDARY SCHOOL

CANDIDATE NAME

CLASS

INDEX NUMBER

PRELIMINARY EXAMINATION 2025
SECONDARY 4 EXPRESS / 5 NORMAL ACADEMIC

MATHEMATICS **4052/02**

Paper 2

25 August 2025

2 hours 15 minutes

Marking Scheme

(Updated 1 Dec 2022 by HOD/Math aligned to Marking Scheme for Specimen Papers 4049)

Question	Answer
1a	$h = 7 + 20t - 5t^2$
	$= -5(t^2 - 4t) + 7$
	$= -5\left[(t-2)^2 - 4\right] + 7$
	$= -5(t-2)^2 + 27$
	maximum height = 27
	OR
	$\frac{dh}{dt} = -10t + 20$
	$-10t + 20 = 0$
	$t = 2$
	When $t = 2$, $h = 7 + 20(2) - 5(2)^2 = 27\text{ m}$
1b	$x^2 + 6x + k = k(x-1)$
	$x^2 + (6-k)x + 2k = 0$
	$(6-k)^2 - 4(1)(2k) < 0$
	$36 - 12k + k^2 - 8k < 0$
	$k^2 - 20k + 32 < 0$
	$(k-2)(k-18) < 0$
	$\therefore 2 < x < 18$
2a	$\tan A = \frac{4}{3}$
	$\tan 2A = \frac{2\left(\frac{4}{3}\right)}{1 - \left(\frac{4}{3}\right)^2}$
	$= -\frac{24}{7}$
Question	**Answer**
2b	$\sin B = \frac{12}{13}$, $\cos B = \frac{5}{13}$
	$\sin\left(B + \frac{3\pi}{2}\right) = \sin B \cos\frac{3\pi}{2} + \sin\frac{3\pi}{2}\cos B$
	$= \frac{12}{13} \times 0 + (-1) \times \frac{5}{13}$
	$= -\frac{5}{13}$
	OR
	$\sin\left(B + \frac{3\pi}{2}\right) = \sin\left(2\pi - \frac{\pi}{2} + B\right)$

	$=\sin\left(-\frac{\pi}{2}+B\right)=-\sin\left(\frac{\pi}{2}-B\right)$
	$=-\cos B=-\frac{5}{13}$
2c	$\cos B=2\cos^2\frac{B}{2}-1$
	$\frac{5}{13}=2\cos^2\frac{B}{2}-1$
	$2\cos^2\frac{B}{2}=\frac{18}{13}$
	$\cos^2\frac{B}{2}=\frac{9}{13}$
	$\cos\frac{B}{2}=\frac{3}{\sqrt{13}}$ or $-\frac{3}{\sqrt{13}}$ (rej)

Question	**Answer**
3a	$\angle PQC = \angle PSQ$ (Alternate segment theorem)
	$\angle PQC = \angle PBC$ (angles in same segment)
	Hence $\angle PBC = \angle PSQ$
	By alternate angles of parallel lines, *BC* is parallel to *QS*
3b	$\angle PQC = \angle PSQ$ (Alternate segment theorem)
	$\angle PMQ = \angle QMS$ (Common angle)
	ΔPQM is similar to ΔQSM (AA Similarity Test)
3c	Since ΔPQM is similar to ΔQSM , $\frac{QM}{SM} = \frac{PM}{QM}$
	$(QM)^2 = PM \times SM$

4a	$\frac{d}{dx}(\cot x)=\frac{d}{dx}\left(\frac{\cos x}{\sin x}\right)$
	$=\frac{\sin x(-\sin x)-\cos x(\cos x)}{\sin^2 x}$
	$=\frac{-\sin^2 x-\cos^2 x}{\sin^2 x}$
	$=-\frac{1}{\sin^2 x}$
	$=-\text{cosec}^2\, x$ (Shown)
	OR
	$\frac{d}{dx}(\cot x)=\frac{d}{dx}(\tan x)^{-1}$
	$=-(\tan x)^{-2}(\sec^2 x)$
	$=-\left(\frac{\sin x}{\cos x}\right)^{-2}\left(\frac{1}{\cos^2 x}\right)$
	$=-\left(\frac{\cos^2 x}{\sin^2 x}\right)\left(\frac{1}{\cos^2 x}\right)$
	$=-\frac{1}{\sin^2 x}$
	$=-\text{cosec}^2\, x$ (Shown)
4b	$\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} -\cot^2 x\, dx=\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} 1-\text{cosec}^2 x\, dx$
	$=[x+\cot x]_{\frac{\pi}{4}}^{\frac{\pi}{3}}$
	$=\left[\frac{\pi}{3}+\cot\frac{\pi}{3}\right]-\left[\frac{\pi}{4}+\cot\frac{\pi}{4}\right]$
	$=\frac{\pi}{3}+\frac{1}{\sqrt{3}}-\frac{\pi}{4}-1$
	$=\frac{\pi}{12}+\frac{1}{\sqrt{3}}-1$ or -0.161

Question	Answer
5a	General term $=\binom{8}{r}x^{8-r}\left(-\frac{k}{x^3}\right)^r$
	$=\binom{8}{r}x^{8-r}\left(-1\right)^r k^r\left(x^{-3}\right)^r$
	$=\binom{8}{r}\left(-1\right)^r k^r x^{8-4r}$
	When $8-4r=0$, $r=2$
	$\binom{8}{2}\left(-1\right)^2 k^2 x^{8-4(2)}=7$
	$28k^2=7$
	$k^2=\frac{1}{4}$
	$k=\frac{1}{2}$
5b	When $8-4r=-4$, $r=3$
	$T_4=\binom{8}{3}\left(-1\right)^3\left(\frac{1}{2}\right)^3 x^{-4}$
	$=-7x^{-4}$
	$\left(1+x^4\right)\left(x-\frac{k}{x^3}\right)^8=\left(1+x^4\right)\left(...+7-7x^{-4}+...\right)$
	$=...+7-7+...=...+0+...$
	There is no constant term in the expansion. (Shown)

Question	Answer
6a	$AM = 2\cos\theta$
	$BM = 2\sin\theta$
	$BC = 4\sin\theta$
	Total area $= (4\sin\theta)^2 + \frac{1}{2}(2\cos\theta)(4\sin\theta)$
	$= 16\sin^2\theta + 4\sin\theta\cos\theta$
6b	$16\sin^2\theta + 4\sin\theta\cos\theta$
	$= 8(2\sin^2\theta) + 2(2\sin\theta\cos\theta)$
	$= 8(1 - \cos 2\theta) + 2(\sin 2\theta)$
	$= 2\sin 2\theta - 8\cos 2\theta + 8$
6c	$R = \sqrt{2^2 + 8^2} = \sqrt{68}$
	$\tan\alpha = \frac{8}{2}$
	$\alpha = 76.0°$
	$2\sin 2\theta - 8\cos 2\theta = \sqrt{68}\sin(2\theta - 76.0°)$
7a	$LHS = \dfrac{\cos^2 x - \sin^2 x}{1 + 2\sin x\cos x}$
	$= \dfrac{\cos^2 x - \sin^2 x}{\sin^2 x + \cos^2 x + 2\sin x\cos x}$
	$= \dfrac{(\cos x + \sin x)(\cos x - \sin x)}{(\cos x + \sin x)^2}$
	$= \dfrac{\cos x - \sin x}{\cos x + \sin x}$
	$= \dfrac{1 - \tan x}{1 + \tan x} = RHS$

Question	Answer
7b	$\dfrac{1-\tan x}{1+\tan x}=\dfrac{2}{3}\tan x$
	$3(1-\tan x)=2\tan x+\tan^2 x$
	$2\tan^2 x+5\tan x-3=0$
	$(2\tan x-1)(\tan x+3)=0$
	$\tan x=\dfrac{1}{2}$ or -3
	$x=0.464$ or 3.61
	$x=1.89$ or 5.03
8a	Quotient: $x^2-\frac{3x}{2}-2$ Divisor: $2x-4$ Dividend: $2x^3-7x^2+2x+3$ $-$ $2x^3-4x^2$ $-3x^2+2x+3$ $-$ $-3x^2+6x$ $3-4x$ $-$ $8-4x$ -5

Question	Answer
8b	$2x^3-7x^2+2x+3=0$
	Let $f(x)=2x^3-7x^2+2x+3$
	When $x=3$, $f(3)=2(3)^3-7(3)^2+2(3)+3=0$
	By factor theorem, $(x-3)$ is a factor of f(x).
	$(x-3)(2x^2-x-1)=0$
	$(x-3)(2x+1)(x-1)=0$
	$x=-\frac{1}{2}$, 1 or 3
8c	$3y^6+2y^4-7y^2+2=0$
	$3+\frac{2}{y^2}-\frac{7}{y^4}+\frac{2}{y^6}=0$
	$3+2\left(\frac{1}{y^2}\right)-7\left(\frac{1}{y^4}\right)+2\left(\frac{1}{y^6}\right)=0$
	$3+2\left(\frac{1}{y^2}\right)-7\left(\frac{1}{y^2}\right)^2+2\left(\frac{1}{y^2}\right)^3=0$
	$\therefore x=\frac{1}{y^2}$, Kun Ye is correct

Question	Answer
9a	Height = $3x^2$
	Base = $2x$
	Area = $\frac{1}{2}(2x)(3x^2) = 3x^3$
9b	$\frac{\mathrm{d}y}{\mathrm{d}x} = 6x$
	$\frac{\mathrm{d}y}{\mathrm{d}t} = \frac{\mathrm{d}y}{\mathrm{d}x} \times \frac{\mathrm{d}x}{\mathrm{d}t}$
	$3 = 6x \times \frac{\mathrm{d}x}{\mathrm{d}t}$
	$3 = 6x \times \frac{\mathrm{d}x}{\mathrm{d}t}$
	$\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{1}{2x}$
	When $x = 4$, $\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{1}{8}$
	Rate of change of PQ = $\frac{1}{4}$ unit/s
9c	$\frac{\mathrm{d}A}{\mathrm{d}x} = 9x^2$
	$\frac{\mathrm{d}A}{\mathrm{d}t} = \frac{\mathrm{d}A}{\mathrm{d}x} \times \frac{\mathrm{d}x}{\mathrm{d}t}$
	$\frac{\mathrm{d}A}{\mathrm{d}t} = 9x^2 \times \frac{1}{8}$
	When $x = 4$, $\frac{\mathrm{d}A}{\mathrm{d}t} = 18$ unit^2/s

Question	**Answer**
10a	y-coordinate of centre = $\dfrac{-1+9}{2}=4$
10b	Let the x-coordinate of the centre be a,
	$(x-a)^2+(y-4)^2=25$
	From the line, $y=\dfrac{4x+9}{3}$
	$(x-a)^2+\left(\dfrac{4x+9}{3}-4\right)^2=25$
	$(x-a)^2+\left(\dfrac{4}{3}x-1\right)^2=25$
	$x^2-2ax+a^2+\dfrac{16}{9}x^2-\dfrac{8}{3}x+1=25$
	$\dfrac{25}{9}x^2+\left(-2a-\dfrac{8}{3}\right)x+a^2-24=0$
	Since the line is a tangent, $b^2-4ac=0$
	$\left(-2a-\dfrac{8}{3}\right)^2-4\left(\dfrac{25}{9}\right)(a^2-24)=0$
	$4a^2+\dfrac{32}{3}a+\dfrac{64}{9}-\dfrac{100}{9}a^2+\dfrac{2400}{9}=0$
	$36a^2+96a+64-100a^2+2400=0$
	$-64a^2+96a+2464=0$
	$2a^2-3a-77=0$
	$(2a+11)(a-7)=0$
	$a=7$ or -5.5 (rej)

Question	Answer
10c	Distance between point and centre $=\sqrt{(7-3)^2+(4-0)^2}$
	$=5.66$ unit
	Since the distance is longer than the radius, the point lies outside.
11a	$CX=\sqrt{(30-x)^2+20^2}$
	$=\sqrt{x^2-60x+1300}$
	$T=\frac{x}{4}+\frac{1}{2}\sqrt{x^2-60x+1300}$
11b	$\frac{dT}{dx}=\frac{1}{4}+\frac{1}{2}\left[\frac{1}{2}\left(x^2-60x+1300\right)^{-\frac{1}{2}}(2x-60)\right]$
	$=\frac{1}{4}+\frac{2x-60}{4\sqrt{x^2-60x+1300}}$
11c	$\frac{1}{4}+\frac{2x-60}{4\sqrt{x^2-60x+1300}}=0$
	$\sqrt{x^2-60x+1300}+2x-60=0$
	$\sqrt{x^2-60x+1300}=60-2x$
	$x^2-60x+1300=3600-240x+4x^2$
	$3x^2-180x+2300=0$
	$x=\frac{180\pm\sqrt{180^2-4(3)(2300)}}{2(3)}$
	$x=41.5$ (rej) or 18.5

Name:	Index No.:	Class:

PRESBYTERIAN HIGH SCHOOL

ADDITIONAL MATHEMATICS **4049/02**
Paper 2

26 August 2025 Tuesday 2 hours 15 min

2025 SECONDARY FOUR EXPRESS / FIVE NORMAL (ACADEMIC) PRELIMINARY EXAMINATIONS

DO NOT OPEN THIS QUESTION PAPER UNTIL YOU ARE TOLD TO DO SO.

INSTRUCTIONS TO CANDIDATES
Write your name, index number and class in the spaces provided above.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Write your answers in the spaces provided below the questions.
Give non-exact numerical answers correct to 3 significant figures or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 90.

For Examiner's Use													
Qn	1	2	3	4	5	6	7	8	9	10	11		*Marks Deducted*
Marks													

Category	Accuracy	Units	Notations	Others
Question No.				

TOTAL MARKS
90

Setter: Ms Sabrina Tan
Vetter: Mr Tan Lip Sing

This question paper consists of **21** printed pages and **1** blank page.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

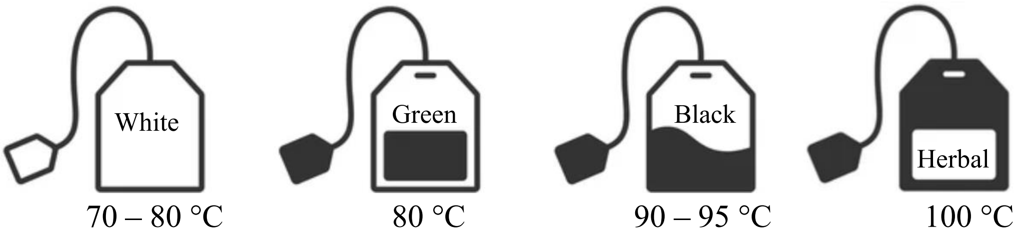
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 The equation of a curve is $y = \dfrac{5x}{6 - 2x}$. Find the x-coordinates of the points at which the gradient of the curve is 7.5. [4]

2 The temperature of water in a tea cup, T °C, t minutes after boiling water is poured into the empty cup can be modelled by the formula $T = 25 + 75\mathrm{e}^{-kt}$.



The diagram above shows the ideal brewing temperatures for various teas at a particular teahouse. At this teahouse, green tea leaves are placed into the tea cup for brewing at the ideal temperature when $t = 6$.

Determine whether the teahouse's standards for brewing tea would be adhered to if black tea leaves are placed into the teacup when $t = 2$. [4]

3 It is given that $f'(x) = 6\cos 3x - 12\sin 3x$ and $f\left(\frac{\pi}{3}\right) = 0$.

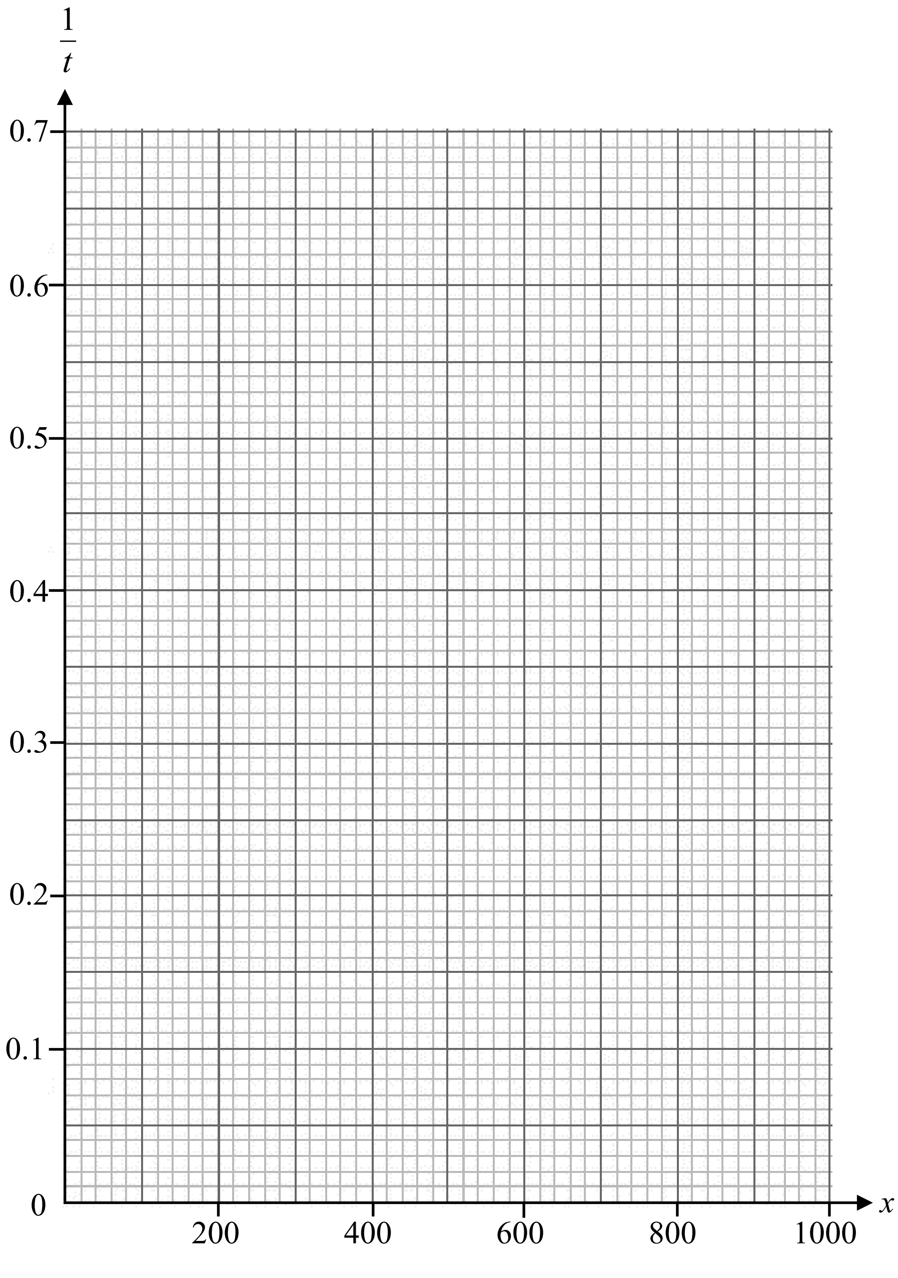
(a) Find f(x). [4]

(b) Find the exact value of $f''\left(\frac{\pi}{12}\right)$. [2]

4 **(a)** The time taken, t in minutes, to download a particular gaming software is related to internet speed, x Mbps. The variables x and t are related by the formula $t = \frac{a}{b+x}$, where a and b are positive constants. The data below shows some measured values of x and t.

x (Mbps)	50	100	200	500	1000
t (minutes)	23	13	7.3	3.2	1.6

(i) Plot $\frac{1}{t}$ against x and draw a straight line graph. [2]



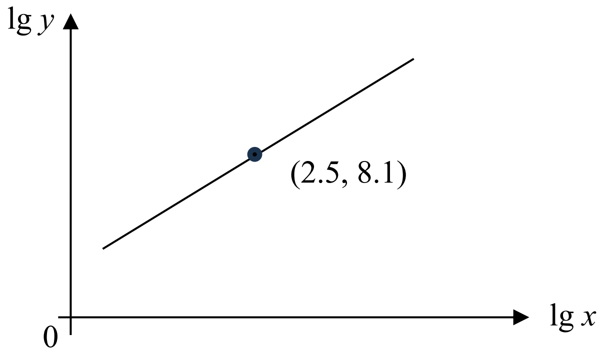
(ii) Use the graph to estimate the value of a and of b, correct to 2 significant figures. [4]

(iii) Use the graph to estimate the time taken for John to download the gaming software if the speed of his internet is 240 Mbps. [2]

(iv) Suggest an alternative set of variables to be plotted on each axis to draw a straight line to represent the formula. [1]

(b) The diagram shows part of a straight line graph drawn to represent the equation $y = px^q$.

Given that the line passes through the point (2.5, 8.1) and has a gradient of 3, find the value of p correct to the nearest integer. [3]



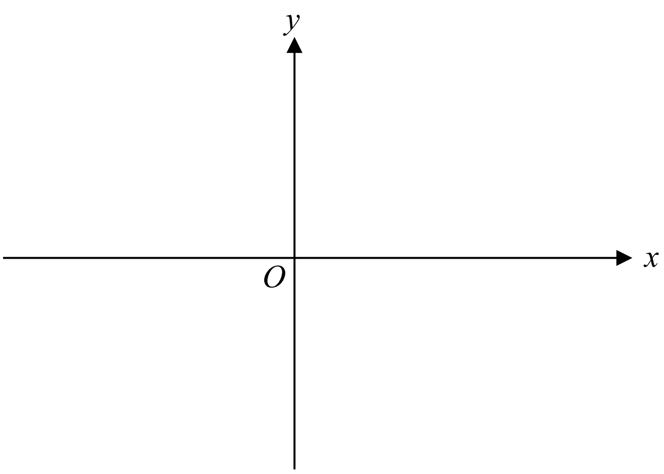
5 **(a)** Find the range of values of the constant *k* for which $k\left(x^2+x\right)+2\left(x^2+x+1\right)=0$ has no real roots. [4]

(b) Hence, explain why $-\left(x^2+x\right)+2\left(x^2+x+1\right)$ is always positive. [2]

6 **(a)** By using a suitable substitution, or otherwise, solve the equation

$$3\log_6 x = \log_x 3 + \log_x 2 + \log_5 25\,.$$ [4]

(b) Sketch the graph of $y = \log_6 x$ on the given axes. Label any axial intercepts. [1]



(c) **(i)** It is given that the equation of a curve is $y = e^{2x+1} + 2e^{x+1} - 3e$.

Explain why the curve has no stationary point. [3]

(ii) Find the x-intercept of the curve $y = e^{2x+1} + 2e^{x+1} - 3e$. [3]

7 **(a)** Factorise $x^3 - 125$. [1]

(b) The expression $-2x^3 + 7x^2 + ax + b$, where a and b are integers, has a factor of $x - 5$ and a remainder of -11 when divided by $2x + 1$. Find the value of a and of b. [4]

(c) Hence, or otherwise, show that the equation $x^3 - 125 = -2x^3 + 7x^2 + ax + b$ has only one real root. [4]

8 The equation of a curve is $y = e^{-2x} \tan x$.

(a) Show that $\dfrac{dy}{dx} = e^{-2x}(1 - \tan x)^2$. [3]

(b) Find, in terms of π, the x-coordinate of the stationary point for $0 < x < \dfrac{\pi}{2}$. [2]

(c) Explain why y is never decreasing. [2]

(d) What does your answer to **part (c)** imply about the stationary point found in **part (b)**? Explain your answer. [2]

9 **(a)** State

(i) the principal value of $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$, [1]

(ii) the values between which the principal value of $\tan^{-1}x$ must lie, [1]

(iii) the range of values of *k* for which the equation $4\sin^2 x = k$ has a solution. [1]

(b) **(i)** Prove the identity $\cot x - \cot x \tan^2 x + \tan x = \dfrac{1 + \cos 2x}{\sin 2x}$. [4]

(ii) Hence solve the equation $\cot x - \cot x \tan^2 x + \tan x = \operatorname{cosec} 2x - 3$ for $0° \le x \le 180°$.

[5]

10 **(a)** Points $P(-2, 0)$, $Q(5, 1)$, $R(6, -6)$ and $S(-1, -7)$ lie on a circle.

PQ and SR are parallel chords of the same length.

Show that the centre of the circle is at $(2, -3)$. [2]

(b) Find the equation of the circle. [3]

(c) A second circle has centre C. It is given that the y-axis is the perpendicular bisector of CR and is also a tangent to the second circle.

(i) State the coordinates of C. [1]

(ii) Hence explain why the x-axis is a tangent to the second circle. [2]

11 A particle moves from point P towards point O. The speed of the particle, t seconds after passing point P, can be modelled by the formula $v = (k - 2t)^n$, where k and n are constants.

(a) Find an expression for the acceleration of the particle in terms of k, t and n. [1]

The particle has a speed of 27 m/s when $t = 3.5$ and comes to instantaneous rest 4.8 metres to the right of point O when $t = 8$.

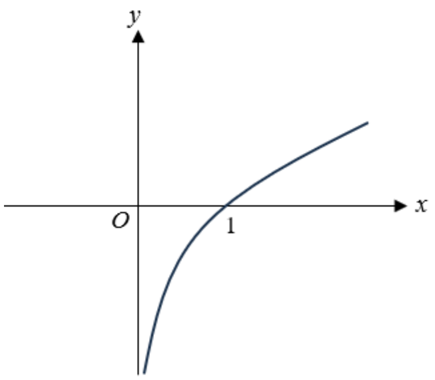
(b) Find the value of k and of n. [3]

(c) Using the values of k and n found in **part (b)**, find the total distance travelled by the particle in the first 8 seconds. [4]

(d) Explain whether the given model can continue to describe the motion of the particle after 8 seconds. [1]

END OF PAPER

Answer Key

1	$x = 2$ or $x = 4$
2	When $t = 2$, $T = 92.6$°C Since the temperature of the water would be within the ideal range of 90 – 95 °C, the standards for brewing black tea **would** be adhered to.
3a	$f(x) = 2\sin 3x + 4\cos 3x + 4$
3b	$-27\sqrt{2}$
4a(ii)	grad=0.0006-0.00062 and a=1600 or 1700 Y-intercept= 0.01, 0.015 or 0.02 and b = 16 – 33
4a(iii)	Accept t = 6.06, 6.25 or 6.45
4a(iv)	xt against t **OR** t against xt
4b	4
5a	$-2 < k < 6$
5b	Since the **discriminant < 0 when k = –1** and the **coefficient of x^2**, i.e. 1, is **positive**, $-(x^2 + x) + 2(x^2 + x + 1)$ is always positive
6a	x = 0.550, 6
6b	
6c(i)	$\frac{dy}{dx} = 2e^{2x+1} + 2e^{x+1}$ Since $e^{2x+1} > 0$ and $e^{x+1} > 0$ for all real values of x, $2e^{2x+1} + 2e^{x+1} > 0$, $\frac{dy}{dx} \neq 0$ Therefore the curve has no stationary points
6c(ii)	$x = 0$
7a	$x^3 - 125 = (x-5)(x^2 + 5x + 25)$
7b	$a = 16$, $b = -5$

8b	$x = \frac{\pi}{4}$
8c	Since $e^{-2x} > 0$ and $(\tan x - 1)^2 \geq 0$ for all real values of x, $\frac{dy}{dx} \geqslant 0$ therefore y is never decreasing.
8d	The stationary point at $x = \frac{\pi}{4}$ is a **point of inflexion.** Since y is never decreasing, the **gradient of the curve must be positive (or dy/dx > 0) slightly to the left and right** of the stationary point.
9a(i)	$-60°$ or $-\frac{\pi}{3}$
9a(ii)	$-90° < \tan^{-1} x < 90°$
9a(iii)	$0 \leq k \leq 4$
9b(ii)	$x = 80.8°, 170.8°$
10b	$(x-2)^2 + (y+3)^2 = 25$ OR $x^2 - 4x + y^2 + 6y - 12 = 0$
10c(i)	(–6, –6)
10c(ii)	Since the y-axis is a tangent to the second circle, the radius of the circle is 6 units. The highest point, H, of the second circle is (–6, 0). Since the circle touches the x-axis at exactly one point at (–6, 0), the x-axis is a tangent to the second circle.
11a	$a = -2n(k-2t)^{n-1}$
11b	$k = 16$, $n = 1.5$
11c	204.8 m
11d	No since when $t > 8$, v and s are undefined.

Name:	Index No.:	Class:

PRESBYTERIAN HIGH SCHOOL

ADDITIONAL MATHEMATICS **4049/01**
Paper 1

25 August 2025 Monday 2 hours 15 min

2025 SECONDARY FOUR EXPRESS / FIVE NORMAL (ACADEMIC) PRELIMINARY EXAMINATIONS

DO NOT OPEN THIS QUESTION PAPER UNTIL YOU ARE TOLD TO DO SO.

INSTRUCTIONS TO CANDIDATES
Write your name, index number and class in the spaces provided above.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Write your answers in the spaces provided below the questions.
Give non-exact numerical answers correct to 3 significant figures or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 90.

	For Examiner's Use													
Qn	1	2	3	4	5	6	7	8	9	10	11	12	13	*Marks Deducted*
Marks														

TOTAL MARKS
/ 90

Category	Accuracy	Units	Notations	Others
Question No.				

Setter: Mr Tan Lip Sing
Vetter: Ms Sabrina Tan

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2\sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

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Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}bc\sin A$$

1 **(a)** Differentiate $\ln\left(\dfrac{3x}{x^2+1}\right)$ with respect to x. [3]

(b) Hence find $\displaystyle\int \frac{2x}{x^2+1}\,\mathrm{d}x$. [2]

2 It is given that $\cos A = \frac{1}{2}$ and $\sin B = -\frac{1}{\sqrt{2}}$ where $0° < A < 90°$ and $180° < B < 270°$.

Find, without using a calculator, the exact value of $\cos(A - B)$, leaving your answer in the form $p\sqrt{2} + q\sqrt{6}$, where p and q are real numbers. [4]

3 Baking powder is poured onto a flat surface at a constant rate of 2π cm^{3}s^{-1}, forming a right circular cone. The radius of the cone is always $\frac{1}{18}$ of its height. Find the rate of change of the radius of the cone after 3 seconds of pouring.

$\left[\text{Volume of cone} = \frac{1}{3}\pi r^2 h\right]$ [5]

4 A and B are the points of intersection of the line $4y = 2x + 1$ and the curve $3y - x = 4xy$.

(a) Find the coordinates of A and of B. [4]

(b) Henry says that the line $2y - 4x = 5$ is perpendicular to the line AB.
Is he correct? Justify your answer with workings. [3]

5 **(a)** Write down and simplify the first three terms in the expansion, in descending powers of x, of $\left(2-\dfrac{3}{x}\right)^8$. [2]

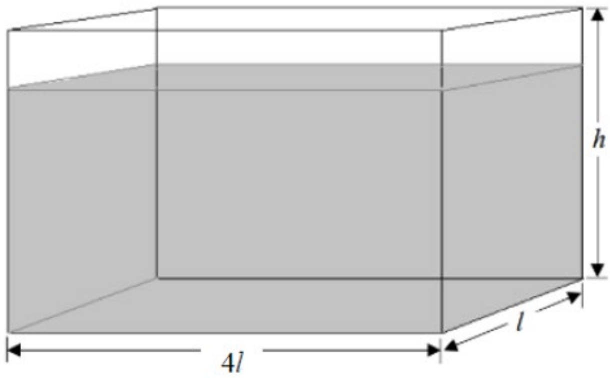
(b) Given that there is no x term in the expansion of $\left(1-2x-kx^2\right)\left(2-\dfrac{3}{x}\right)^8$, find the constant term in the expansion. [4]

6 **(a)** Express $y = 4x - 4x^2 - 3$ in the form $p(x+q)^2 + r$ where p, q and r are constants. [2]

(b) Hence, explain whether $4x - 4x^2 - 3 = 0$ has any real solutions. [2]

TURN OVER FOR QUESTION 7

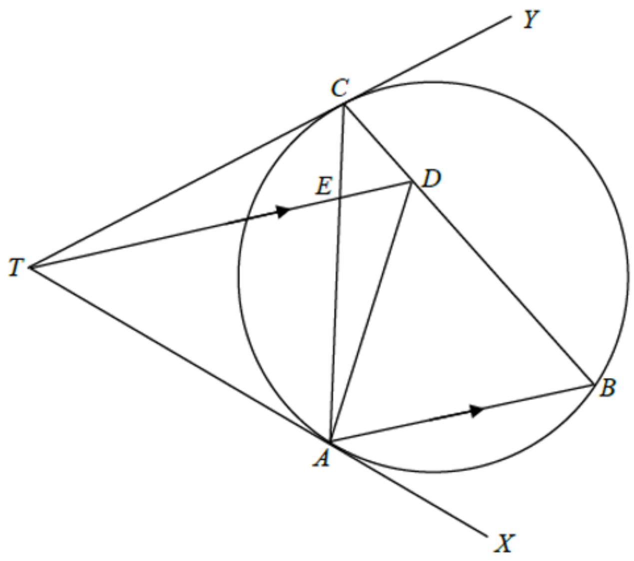
7 Peter constructed an open fish tank with a rectangular base of length $4l$ m, breadth l m, and height h m. He wanted the total outer surface area of the fish tank to be $5\ \text{m}^2$.



(a) Show that the volume of the tank, $V\ \text{m}^3$, is given by $V = \frac{2}{5}\left(5l - 4l^3\right)$. [3]

(b) Determine the area of the rectangular base for the tank to contain the maximum amount of water when filled to the brim. [4]

8 In the diagram below, *TAX* and *TCY* are tangents to the circle at *A* and *C* respectively. *AC* meets *TD* at *E* and *D* is on *BC* such that *TD* is parallel to *AB*.



(a) Prove that angle *ACB* is equal to angle *ATD*. [2]

(b) Using the result from **part (a)**, explain whether a circle can be drawn passing through the points *T*, *A*, *D* and *C*. [2]

(c) Prove that $CE \times EA = DE \times TE$. [3]

9 **(a)** A square has an area $\left(17-8\sqrt{2}\right)\text{ cm}^2$. The length of each side of the square can be expressed in the form $\left(a+b\sqrt{2}\right)\text{ cm}$, where a and b are integers. Show that $2b^4-17b^2+16=0$. [4]

(b) [The area of a sector is $\frac{1}{2}r^2\theta$ and the arc length of a sector is $r\theta$.]

The sector of a circle with radius, *r*, has an arc length of $\left(\sqrt{15}-\sqrt{3}\right)$ cm and an area of $\left(3\sqrt{3}-\sqrt{15}\right)$ cm^2. Show that $r=\dfrac{6\sqrt{3}-2\sqrt{15}}{\sqrt{15}-\sqrt{3}}$ and hence express *r* in the form $\left(p+q\sqrt{5}\right)$ cm, where *p* and *q* are integers. [5]

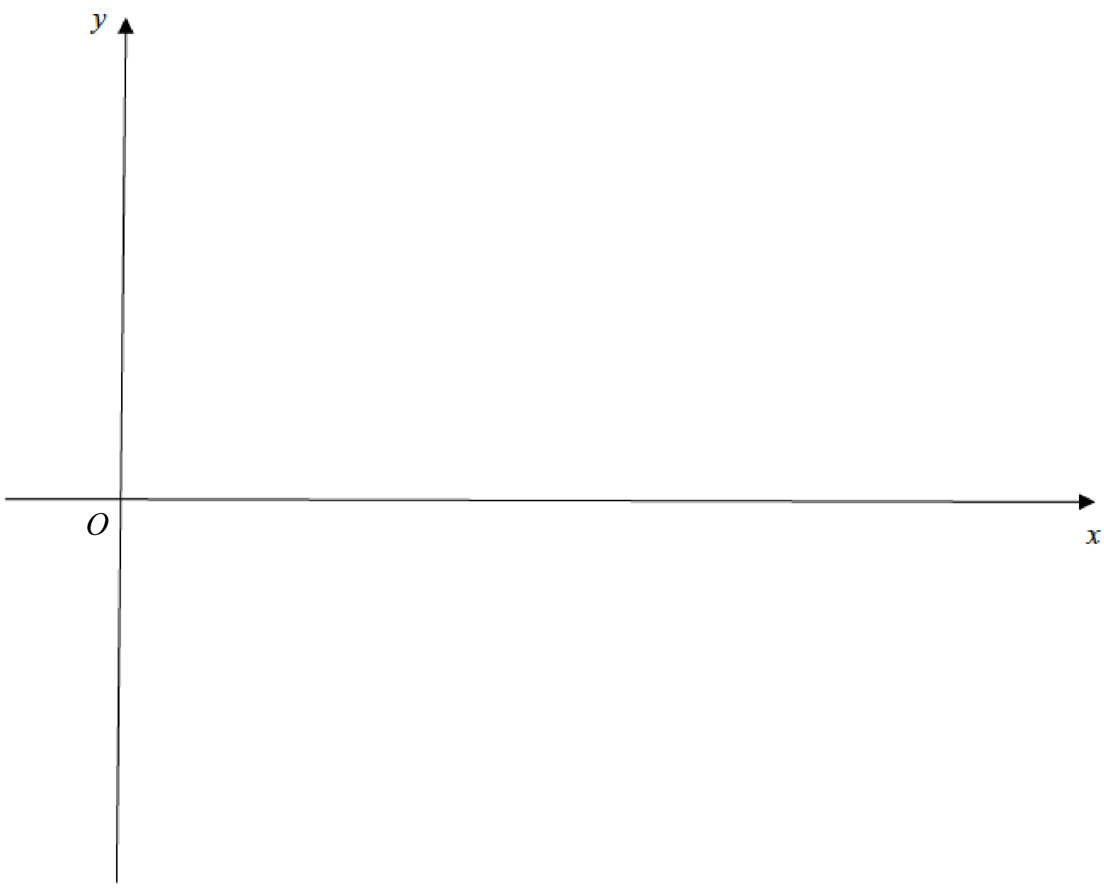
10 It is given that $f(x) = 2\sin\frac{x}{2}$ and $g(x) = 3\cos x + 1$, where $0 \le x \le 2\pi$.

(a) State the period of $f(x)$. [1]

(b) State the smallest value of $f(x)$. [1]

(c) State the largest value of $g(x)$. [1]

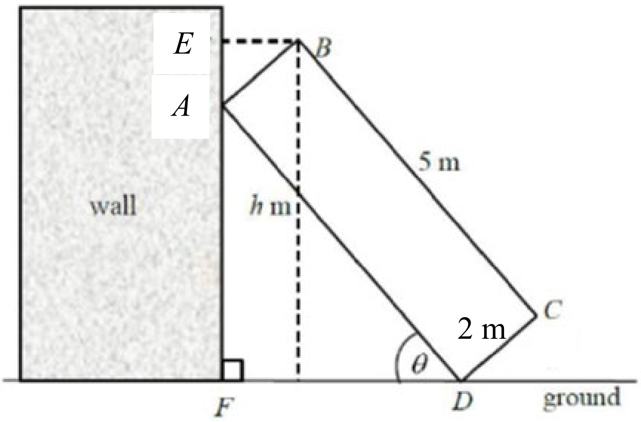
(d) Sketch on the same axes, the graphs of $y = f(x)$ and $y = g(x)$ for $0 \le x \le 2\pi$.
Label your graphs clearly. [4]



(e) The solutions to the equation $f(x) = g(x)$ for $0 \le x \le 2\pi$ are a and b, where $a < b$.
State, in terms of a and b, the range of values of x for which $f(x) > g(x)$. [1]

11 The diagram shows a rectangular wooden plank *ABCD*, 5 m by 2 m, leaning against a vertical wall. *AD* makes an acute angle θ with the horizontal ground.

E is a point on the vertical wall such that *EB* is parallel to the ground and *F* is a point on the ground such that *AF* is perpendicular to *FD*.
The point *B* is *h* m vertically above the ground.



(a) Show that $h = 5\sin\theta + 2\cos\theta$. [2]

(b) Express *h* in the form $R\sin(\theta + \alpha)$, where $R > 0$, and $0° < \alpha < 90°$. [3]

(c) Find the greatest possible value of h and the value of θ at which it occurs. [2]

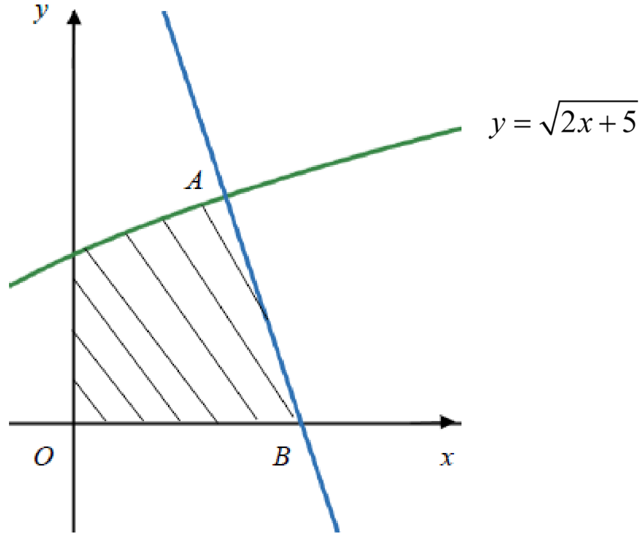
(d) Find the value of θ for which $h = \sqrt{15}$ m. [2]

12 **(a)** By using long division, divide $4x^3+5x^2+x-1$ by $x^2(x+1)$. [1]

(b) Hence, express $\dfrac{4x^3+5x^2+x-1}{x^2(x+1)}$ in partial fractions. [5]

(c) Hence, find $\int \frac{4x^3+5x^2+x-1}{x^2(x+1)}\,\mathrm{d}x$. [3]

13 The diagram shows part of the curve $y = \sqrt{2x+5}$. A is a point on the curve and the x-coordinate of A is 2. The normal to the curve at A meets the x-axis at B.



(a) Find the equation of the normal to the curve at A. [5]

(b) Find the area of the shaded region bounded by the normal AB, the curve, the x-axis and the y-axis. [5]

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2025 PRELIM
CHUNG CHENG HIGH (YISHUN)
ADDITIONAL MATH (G3)
PAPER 1

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

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Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

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$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
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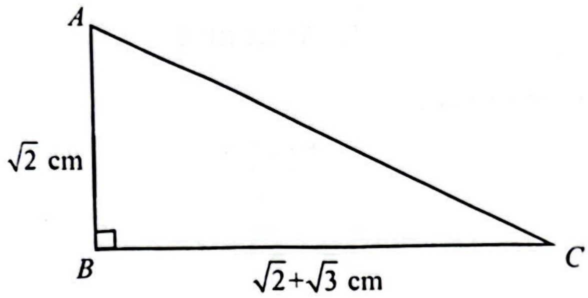
1



The diagram above shows a right-angled triangle ABC, where $AB=\sqrt{2}$ cm, and $BC=\sqrt{2}+\sqrt{3}$ cm.

(a) Find the exact value of $\tan\angle ACB$ in the form of $a+\sqrt{b}$, where a and b are integers. [2]

(b) Hence, find the exact value of $\sec^2\angle ACB$ in the form $m+n\sqrt{k}$, where m and n are integers. [2]

2 A container has a capacity of 840 cm^3 and is initially filled completely with water. The volume, V cm^3, of water in the container is given by $V = h^2 + 2h$, where h cm is the height of the water level in the container. Due to leakage at the bottom of the container, the height of the water level in the container decreases at a rate of $\frac{5t}{3}$ cm/s, where t is the time in seconds.

(a) Find the initial height of the water level in the container. [2]

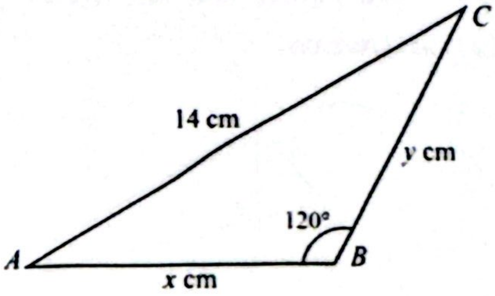
(b) Show that the height of the water level h can be expressed as $h = -\frac{5t^2}{6} + 28$. [2]

(c) Find the rate of decrease of volume when $t = 3$. [5]

3 (a) Find $\frac{\mathrm{d}}{\mathrm{d}x}\left(xe^{\frac{1}{2}x}\right)$. [2]

(b) Hence evaluate $\int_0^4 xe^{\frac{1}{2}x}\,\mathrm{d}x$, leaving your answer in the form $k\left(e^2+1\right)$, where k is a constant to be found. [5]

4 In triangle ABC below, $AB = x$ cm, $BC = y$ cm, $AC = 14$ cm and angle $ABC = 120°$.



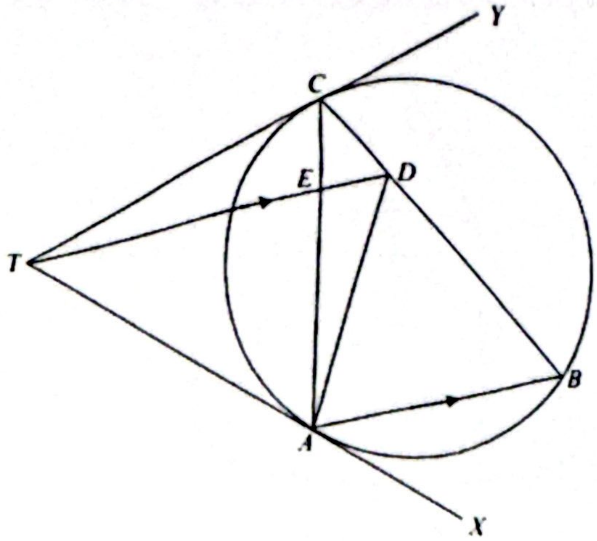
(a) Using cosine rule, form an equation involving x and y. [1]

(b) Given that the perimeter of triangle ABC is 30 cm, find the exact area of triangle ABC. [4]

5 **(a)** By considering the general term in the expansion of $\left(x^5+\frac{1}{x}\right)^7$, explain why there are no even powers of x in its expansion. [2]

(b) Given that the coefficient of x^6 in the expansion of $\left(x^5+\frac{1}{x}\right)^7+(kx+3)^7$ is 1344, where k is a positive constant, find the coefficient of x^4 in the expansion. [4]

6 In the diagram below, TAX and TCY are tangents to the circle at A and C respectively. AC meets TD at E. D is on BC such that TD is parallel to AB.

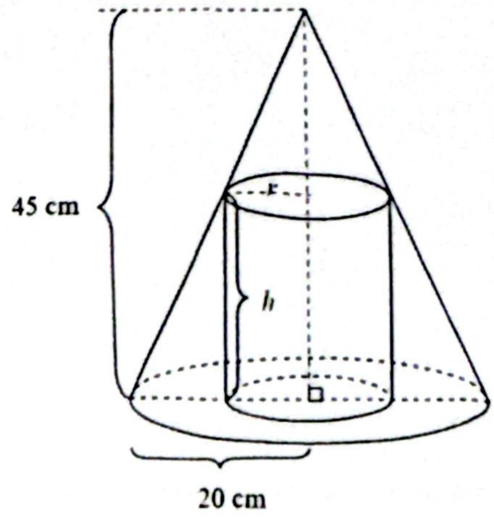


(a) Prove that angle ACB is equal to angle ATD. [2]

(b) Explain why a circle can be drawn passing through the points T, A, D and C. [1]

(c) Hence prove that $CE \times AE = DE \times TE$. [3]

7 The diagram shows a solid cylinder of radius r cm and height h cm inscribed in a hollow cone of height 45 cm and base radius 20 cm. The cylinder rests on the base of the cone and the circumference of the top surface of the cylinder touches the curved surface of the cone.



(a) Show that the volume, V cm^3, of the cylinder is given by $V = 45\pi r^2 - \frac{9}{4}\pi r^3$. [3]

(b) Given that r can vary, find the maximum volume of the cylinder, leaving your answer in terms of π. [5]

(c) Hence show that the cylinder occupies at most $\frac{4}{9}$ of the volume of the cone. [2]

8 **(a)** Find the value of a and of b for which $\{x : x < -3.5 \text{ or } x > 2\}$ is the solution set of $b < x^2 + ax$. [3]

(b) Find the range of values of p for which $y = x^2 - px - 3x - p$ is always positive. [3]

(c) Explain whether the line $y = -5x - 2$ intersects the curve $y = kx^2 + 3$ where $k < 1$. [3]

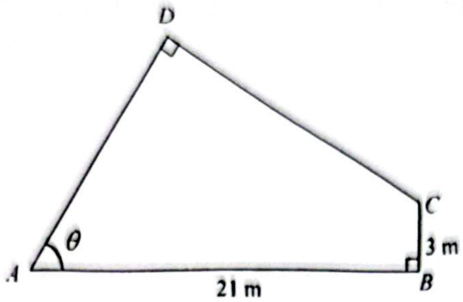
9 (a) Express $5\sin^2 x - \cos^2 x - 1$ in the form $1 - k\cos 2x$. [2]

(b) State the amplitude and period, in radians, of $5\sin^2 x - \cos^2 x - 1$. [2]

(c) Sketch the graph of $y = 5\sin^2 x - \cos^2 x - 1$ for $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$. [3]

(d) By drawing the line $y = 1 - \frac{3x}{2\pi}$ on the same axes, state the number of solutions to the equation $2\pi - 3x = 2\pi\left(5\sin^2 x - \cos^2 x - 1\right)$ in the range $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$. [2]

10



The diagram shows a quadrilateral field $ABCD$, where $AB = 21$ m and $BC = 3$ m.
Angle ABC = angle $ADC = 90°$. Angle $BAD = \theta$, for $0° < \theta < 90°$, and can vary.
The perimeter of the fencing around the quadrilateral field $ABCD$ is P m.

(a) Show that $P = 24 + 18\cos\theta + 24\sin\theta$. [3]

(b) Express P in the form $R\sin(\theta + \alpha) + 24$, where $R > 0$ and $0° < \alpha < 90°$. [3]

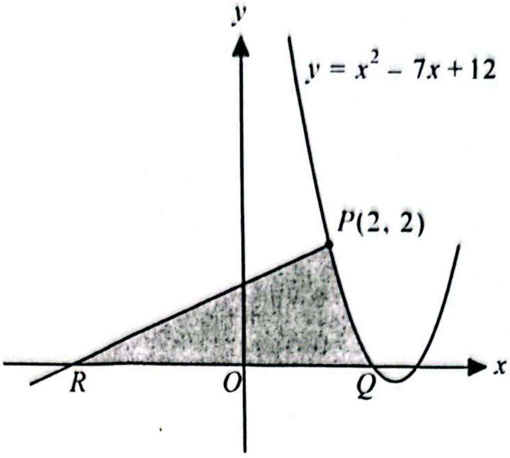
(c) Given that the total perimeter of the fencing is 53 m, find the value(s) of θ. [2]

(d) Explain why the total length of the fencing will never exceed a certain value and state this value. [2]

11 **(a)** Express $\dfrac{11x+12}{x^2(x+4)}$ in partial fractions. [5]

(b) Hence find $\displaystyle\int \frac{11x+12}{x^2(x+4)}\,dx$. [2]

12 The diagram shows part of the curve $y = x^2 - 7x + 12$, cutting the x-axis at Q. The normal to the curve at $P(2, 2)$ meets the x-axis at R.



Show that the area of the shaded region bounded by the x-axis, the line PR and the curve is $6\frac{5}{6}$ units^2. [8]

中正中学 义顺

CHUNG CHENG HIGH SCHOOL (YISHUN)

2025 Preliminary Examination

Secondary Four Express / Five Normal Academic

CANDIDATE NAME				
FORM CLASS / SUBJECT GROUP	/		INDEX NUMBER	

ADDITIONAL MATHEMATICS — 4049/01

Paper 1

21 August 2025

2 hours 15 minutes

Candidates answer on the Question Paper.

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.
Up to 2 marks may be deducted for improper presentation.

The number of marks is given in brackets [ ] at the end of each question or part question.

Question Number	Marks Possible	Marks Obtained
1	4	
2	9	
3	7	
4	5	
5	6	
6	6	
7	10	
8	9	
9	9	
10	10	
11	7	
12	8	
Presentation Deduction		– 1 / – 2
TOTAL	90	

This document consists of **17** printed pages and **1** blank page.

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$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

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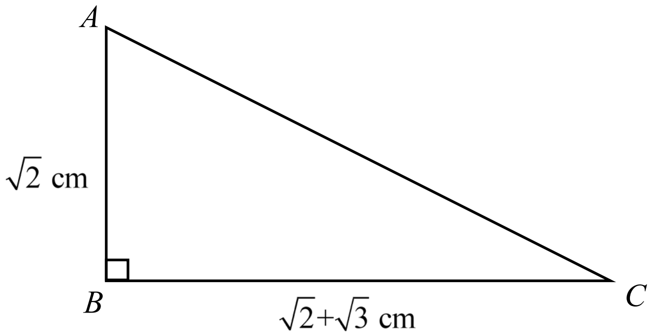
1



The diagram above shows a right-angled triangle ABC, where $AB=\sqrt{2}$ cm, and $BC=\sqrt{2}+\sqrt{3}$ cm.

(a) Find the exact value of $\tan\angle ACB$ in the form of $a+\sqrt{b}$, where a and b are integers. [2]

$$\tan\angle ACB=\frac{\sqrt{2}}{\sqrt{2}+\sqrt{3}}\times\frac{\sqrt{2}-\sqrt{3}}{\sqrt{2}-\sqrt{3}}$$
$$=\frac{2-\sqrt{6}}{2-3}$$
$$=-2+\sqrt{6}$$

(b) Hence, find the exact value of $\sec^2\angle ACB$ in the form $m+n\sqrt{k}$, where m and n are integers. [2]

$$\sec^2 A=1+\tan^2 A$$
$$=1+(\sqrt{6}-2)^2$$
$$=1+6-4\sqrt{6}+4$$
$$=11-4\sqrt{6}$$

2 A container has a capacity of 840 cm^3 and is initially filled completely with water. The volume, V cm^3, of water in the container is given by $V = h^2 + 2h$ where h cm is the height of the water level in the container. Due to leakage at the bottom of the container, the height of the water leve in the container decreases at a rate of $\frac{5t}{3}$ cm/s, where t is the time in seconds.

(a) Find the initial height of the water level in the container. [2]

at $t = 0,\ V = 840$

$h^2 + 2h = 840$

$h^2 + 2h - 840 = 0$

$(h-28)(h+30) = 0$

$h = 28$ or $h = -30$ (rejected)

$\therefore$ Initial Height = 28 cm

(b) Show that the height of the water level h can be expressed as $h = -\frac{5t^2}{6} + 28$. [2]

$h = \int -\frac{5t}{3}\,\mathrm{d}t$

$= -\frac{5t^2}{6} + c$

when $t = 0, h = 28,$

$28 = 0 + c$

$c = 28$

$\therefore h = -\frac{5t^2}{6} + 28$ (shown)

(c) Find the rate of decrease of volume when $t = 3$. [5]

$\frac{\mathrm{d}V}{\mathrm{d}h} = 2h + 2$

$\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{-5(3)}{3}$

$= -5$

when $t = 3,$

$h = \frac{-5(3)^2}{6} + 28$

$= 20.5$

$\frac{\mathrm{d}V}{\mathrm{d}h} = 2(20.5) + 2$

$= 43$

$\frac{\mathrm{d}V}{\mathrm{d}t} = \frac{\mathrm{d}v}{\mathrm{d}h} \times \frac{\mathrm{d}h}{\mathrm{d}t}$

$= 43 \times (-5)$

$= -215\ \text{cm}^3/\text{s}$

Rate of decrease = $215\text{cm}^3/\text{s}$

3 **(a)** Find $\frac{d}{dx}\left(xe^{\frac{1}{2}x}\right)$. [2]

$\frac{d}{dx}\left(xe^{\frac{1}{2}x}\right) = xe^{\frac{1}{2}x}\left(\frac{1}{2}\right) + e^{\frac{1}{2}x}$

$= \frac{1}{2}xe^{\frac{1}{2}x} + e^{\frac{1}{2}x}$

Accept $e^{\frac{1}{2}x}\left(\frac{1}{2}x + 1\right)$

(b) Hence evaluate $\int_0^4 xe^{\frac{1}{2}x}\,dx$, leaving your answer in the form $k\left(e^2+1\right)$, where k is a constant to be found. [5]

$\int_0^4 \frac{1}{2}xe^{\frac{1}{2}x} + e^{\frac{1}{2}x}\ dx = \left[xe^{\frac{1}{2}x}\right]_0^4$

$\int_0^4 xe^{\frac{1}{2}x} + 2e^{\frac{1}{2}x}\ dx = 2\left[xe^{\frac{1}{2}x}\right]_0^4$

$\int_0^4 xe^{\frac{1}{2}x}\ dx = 2\left[xe^{\frac{1}{2}x}\right]_0^4 - \int_0^4 2e^{\frac{1}{2}x}dx$

$= 2\left[xe^{\frac{1}{2}x}\right]_0^4 - \left[4e^{\frac{1}{2}x}\right]_0^4$

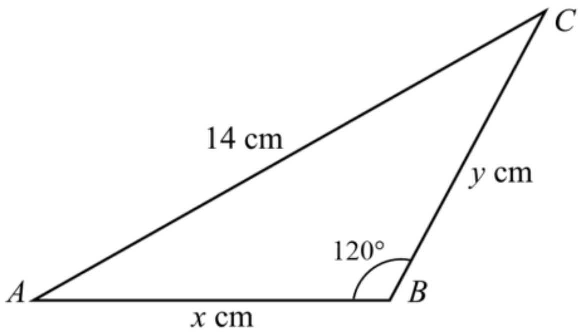
$= 2\left[4e^2 - 0\right] - \left[4e^2 - 4\right]$

$= 8e^2 - 4e^2 + 4$

$= 4e^2 + 4$

$= 4(e^2 + 1)$

4 In triangle ABC below, $AB = x$ cm, $BC = y$ cm, $AC = 14$ cm and angle $ABC = 120^\circ$.



(a) Using cosine rule, form an equation involving x and y. [1]

$14^2 = x^2 + y^2 - 2xy\cos 120^\circ$

$196 = x^2 + y^2 + xy$

(b) Given that the perimeter of triangle ABC is 30 cm, find the exact area of triangle ABC. [4]

$x + y + 14 = 30$

$x = 16 - y$ ---------- [1]

$196 = x^2 + y^2 + xy$ ------[2]

sub [1] into [2]

$196 = (16 - y)^2 + y^2 + (16 - y)y$

$196 = 256 - 32y + y^2 + y^2 + 16y - y^2$

$y^2 - 16y + 60 = 0$

$(y - 6)(y - 10) = 0$

$y = 6$ or $y = 10$

$x = 10$ or $x = 6$

$$\text{Area of } \Delta ABC = \frac{1}{2}(10)(6)\sin 120^\circ$$

$$= 30\left(\frac{\sqrt{3}}{2}\right)$$

$$= 15\sqrt{3} \text{ cm}^2$$

5 **(i)** By considering the general term in the expansion of $\left(x^5+\frac{1}{x}\right)^7$, explain why there are no even powers of x in its expansion. [2]

$$T_{r+1}=\binom{7}{r}\left(x^5\right)^{7-r}\left(\frac{1}{x}\right)^r$$
$$=\binom{7}{r}(x)^{35-5r}\,x^{-r}$$
$$=\binom{7}{r}x^{35-6r}$$

$6r$ is always even by all non-integers of r.
$35-6r$ is always odd.

$\therefore$ There are no even powers of x in its expansion.

(ii) Given that the coefficient of x^6 in the expansion of $\left(x^5+\frac{1}{x}\right)^7+(kx+3)^7$ is 1344, where k is a positive constant, find the coefficient of x^4 in the expansion. [4]

$$T_{r+1}=\binom{7}{r}(kx)^{7-r}\,3^r$$
$$=\binom{7}{r}(k)^{7-r}\,3^r\,x^{7-r}$$

$7-r=6$
$r=1$

$$T_2=\binom{7}{1}(k)^6\,3^1\,x^6$$

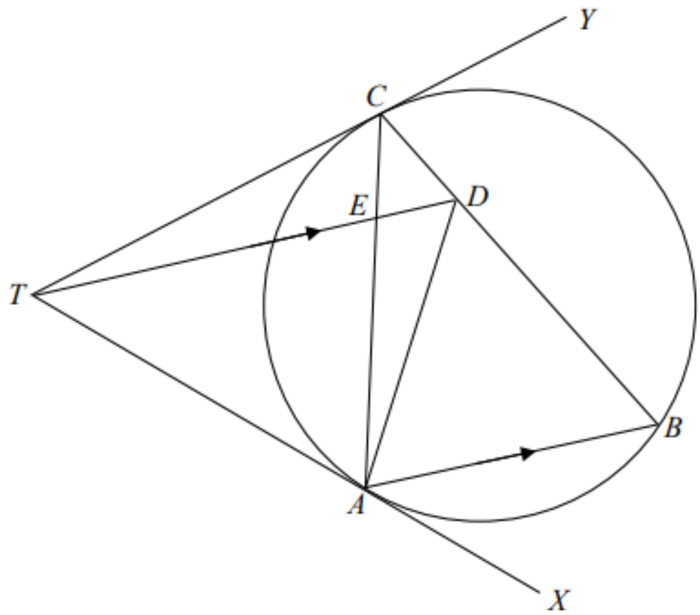
$21k^6=1344$
$k=2$ or $k=-2$ (rejected)

$7-r=4$
$r=3$

$$T_4=\binom{7}{3}(2)^4\,3^3\,x^4$$

$\therefore$ coefficient of $x^4=15120$

6 In the diagram below, *TAX* and *TCY* are tangents to the circle at *A* and *C* respectively. *AC* meets *TD* at *E*. *D* is on *BC* such that *TD* is parallel to *AB*.



(i) Prove that angle *ACB* is equal to angle *ATD*. [2]

$\angle ATD = \angle XAB$ (corr angles, $AB // TD$)

$= \angle ACB$ (alternate segment theorem)

(ii) Explain why a circle can be drawn passing through the points T, A, D and C. [1]

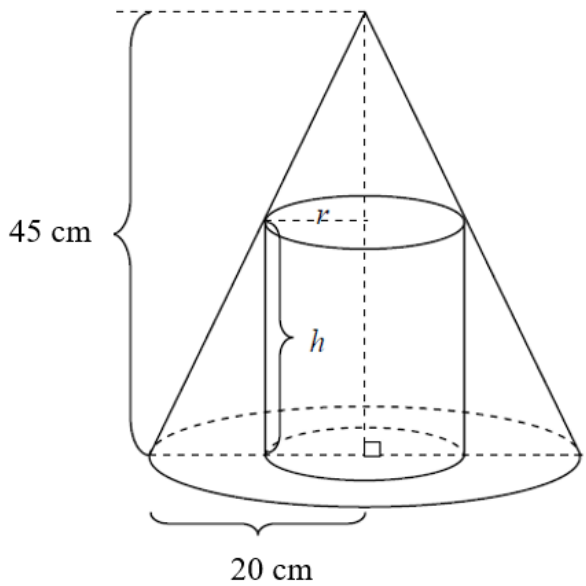
Since $\angle ATD = \angle ACB$, they fulfil the property of angles in the same segment.
$\therefore$ A circle can be drawn passing through the points T, A, D and C.

(iii) Hence prove that $CE \times AE = DE \times TE$. [3]

$\angle ATE = \angle DCE$ [from part (a)]
$\angle TEA = \angle CED$ [vert opp $\angle$]
$\therefore \Delta ATE$ is similar to ΔDCE (AA similarity test)

$\dfrac{TE}{CE} = \dfrac{EA}{ED}$ (Ratio of corr sides of similar triangles are equal)
$CE \times AE = DE \times TE$ (proven)

7 The diagram shows a solid cylinder of radius r cm and height h cm inscribed in a hollow cone of height 45 cm and base radius 20 cm. The cylinder rests on the base of the cone and the circumference of the top surface of the cylinder touches the curved surface of the cone.



(i) Show that the volume, V cm^3, of the cylinder is given by $V = 45\pi r^2 - \frac{9}{4}\pi r^3$. [3]

$$\frac{45-h}{45} = \frac{r}{20}$$
$$20(45-h) = 45r$$
$$900 - 20h = 45r$$
$$h = \frac{900-45r}{20}$$
$$= 45 - \frac{9}{4}r$$
$$V = \pi r^2 h$$
$$= \pi r^2 (45 - \frac{9}{4}r)$$
$$= 45\pi r^2 - \frac{9}{4}\pi r^3 \text{ (shown)}$$

(ii) Given that r can vary, find the maximum volume of the cylinder, leaving your answer in terms of π. [5]

$$\frac{dV}{dr} = 90\pi r - \frac{27}{4}\pi r^2$$

when V is max,

$$90\pi r - \frac{27}{4}\pi r^2 = 0$$

$$\pi r\left(90 - \frac{27}{4}r\right) = 0$$

$r = 0$ or $r = 13\frac{1}{3}$

(rejected)

$$\frac{d^2V}{dr^2} = 90\pi - \frac{27}{2}\pi r$$

when $r = \frac{40}{3}$,

$$\frac{d^2V}{dr^2} = 90\pi - \frac{27}{2}\pi\left(\frac{40}{3}\right)$$

$$= -282.74$$

$$< 0$$

Since $\frac{d^2V}{dr^2} < 0$, $V = \frac{8000\pi}{3}\text{cm}^3$ is a maximum volume.

(iii) Hence show that the cylinder occupies at most $\frac{4}{9}$ of the volume of the cone. [2]

$$\text{Volume of cone} = \frac{1}{3}\pi(20)^2(45)$$

$$= 6000\pi$$

$$\therefore \text{ratio} = \frac{\frac{8000\pi}{3}}{6000\pi}$$

$$= \frac{4}{9} \text{ (shown)}$$

8 **(a)** Find the value of a and of b for which $\{x : x < -3.5 \text{ or } x > 2\}$ is the solution set of $b < x^2 + ax$. [3]

$(2x+7)(x-2) > 0$

$2x^2 + 3x - 14 > 0$

$x^2 + ax - b > 0$

$2x^2 + 2ax - 2b > 0$

$2a = 3$

$a = 1.5$

$2b = 14$

$b = 7$

$x^2 + 1.5x > 7$

(b) Find the range of values of p for which $y = x^2 - px - 3x - p$ is always positive. [3]

$b^2 - 4ac < 0$

$(-p-3)^2 - 4(1)(-p) < 0$

$p^2 + 6p + 9 + 4p < 0$

$p^2 + 10p + 9 < 0$

$(p+1)(p+9) < 0$

$-9 < p < -1$

(c) Explain whether the line $y = -5x - 2$ intersects the curve $y = kx^2 + 3$ where $k < 1$. [3]

$-5x - 2 = kx^2 + 3$

$kx^2 + 5x + 5 = 0$

$b^2 - 4ac = 25 - 4k(5)$

$= 25 - 20k$

$k < 1$

$-20k > -20$

$25 - 20k > 5$

> 0

Since $b^2 - 4ac > 0$, for $k < 1$, the line intersects the curve.

9 **(i)** Express $5\sin^2 x - \cos^2 x - 1$ in the form $1 - k\cos 2x$. [2]

$$5\sin^2 x - \cos^2 x - 1 = 5\left(\frac{1-\cos 2x}{2}\right) - \left(\frac{\cos 2x + 1}{2}\right) - 1$$
$$= \frac{5}{2} - \frac{5\cos 2x}{2} - \frac{1}{2}\cos 2x - \frac{1}{2} - 1$$
$$= 1 - 3\cos 2x$$

(ii) State the amplitude and period, in radians, of $5\sin^2 x - \cos^2 x - 1$. [2]

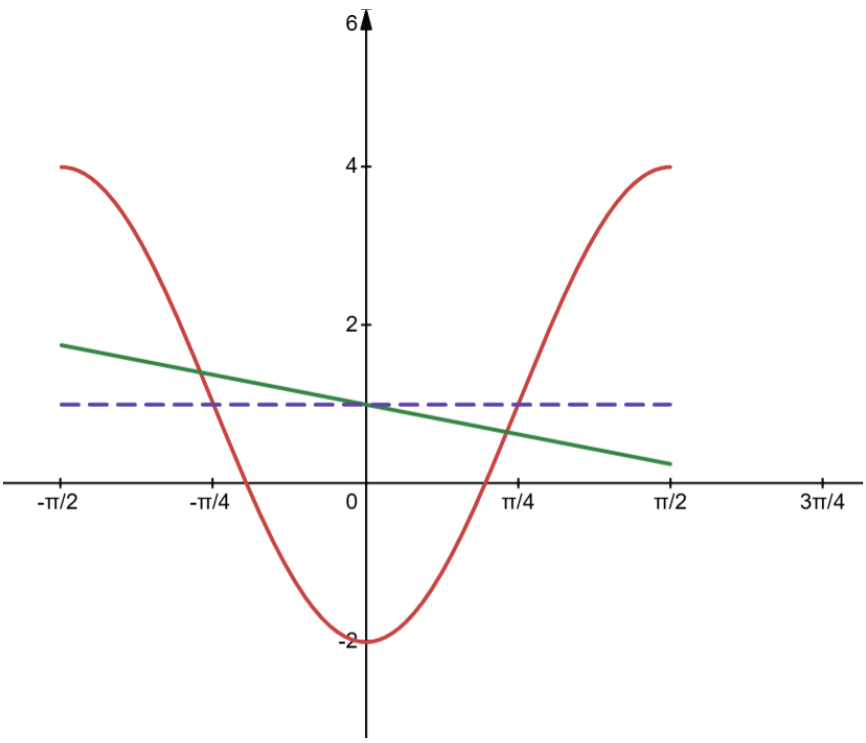
$5\sin^2 x - \cos^2 x - 1 = 1 - 3\cos 2x$

Amplitude $= 3$

Period $= \dfrac{2\pi}{2}$

$= \pi$ radian

(iii) Sketch the graph of $y = 5\sin^2 x - \cos^2 x - 1$ for $-\dfrac{\pi}{2} \le x \le \dfrac{\pi}{2}$. [3]

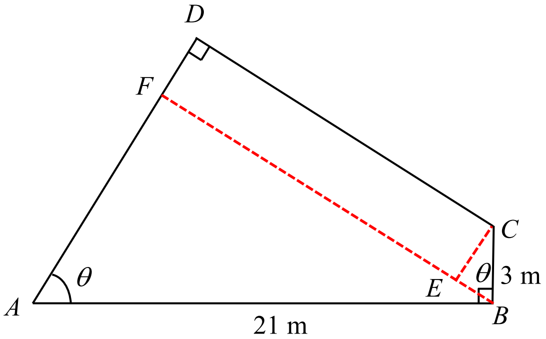


(iv) By drawing the line $y = 1 - \dfrac{3x}{2\pi}$ on the same axes, state the number of solutions to the equation $2\pi - 3x = 2\pi\left(5\sin^2 x - \cos^2 x - 1\right)$ in the range $-\dfrac{\pi}{2} \le x \le \dfrac{\pi}{2}$. [2]

$y = -\dfrac{3}{2\pi}x + 1$

$\therefore$ Number of solutions = 2.

10



The diagram shows a quadrilateral field *ABCD*, where AB = 21 m, BC = 3 m and angle ABC = angle $ADC = 90°$. Angle $BAD = \theta$ and can vary. The perimeter of the fencing around the quadrilateral field *ABCD* is *P* m.

(i) Show that $P = 24 + 18\cos\theta + 24\sin\theta$. [3]

$$\sin\theta = \frac{BF}{21}$$
$$BF = 21\sin\theta$$
$$\cos\theta = \frac{EB}{3}$$
$$EB = 3\cos\theta$$
$$DC = 21\sin\theta - 3\cos\theta$$
$$\cos\theta = \frac{AF}{21}$$
$$AF = 21\cos\theta$$
$$\sin\theta = \frac{EC}{3}$$
$$EC = 3\sin\theta$$
$$AD = 21\cos\theta + 3\sin\theta$$

Hence, $P = 21 + 3 + 21\sin\theta - 3\cos\theta + 21\cos\theta + 3\sin\theta$
$= 24 + 18\cos\theta + 24\sin\theta$ (shown)

(ii) Express P in the form $R\sin(\theta+\alpha)+24$, where $R>0$ and $0°<\alpha<90°$. [3]

$$R=\sqrt{18^2+24^2}$$
$$=\sqrt{900}$$
$$=30$$
$$\tan\alpha=\frac{18}{24}$$
$$=36.8699°$$
$$=36.9°$$

$$\therefore P=30\sin(\theta+36.9°)+24$$

(iii) Given that the total perimeter of the fencing is 53 m, find the value(s) of θ. [2]

$$30\sin(\theta+36.8699°)+24=53$$
$$\sin(\theta+36.8699°)=\frac{29}{30}$$
$$\text{Basic angle}=\sin^{-1}\left(\frac{29}{30}\right)$$
$$=75.1649°$$

$$\theta+36.8699°=75.1649°,104.8351°$$
$$\theta=38.3°,\ 68.0°$$

(iv) Explain why the total length of the fencing will never exceed a certain value and state this value. [2]

$$-1\le\sin(\theta+36.9°)\le1$$
$$-30\le30\sin(\theta+36.9°)\le30$$
$$-6\le30\sin(\theta+36.9°)+24\le54$$

The total length will not exceed 54m as the max length is 54m.

11 **(i)** Express $\dfrac{11x+12}{x^2(x+4)}$ in partial fractions. [5]

$$\frac{11x+12}{x^2(x+4)}=\frac{A}{x}+\frac{B}{x^2}+\frac{C}{x+4}$$

$$11x+12=Ax(x+4)+B(x+4)+Cx^2$$

when $x=0$,

$$12=0+4B+0$$

$$B=3$$

when $x=-4$,

$$-32=0+0+16C$$

$$C=-2$$

when $x=1$,

$$23=5A+15-2$$

$$A=2$$

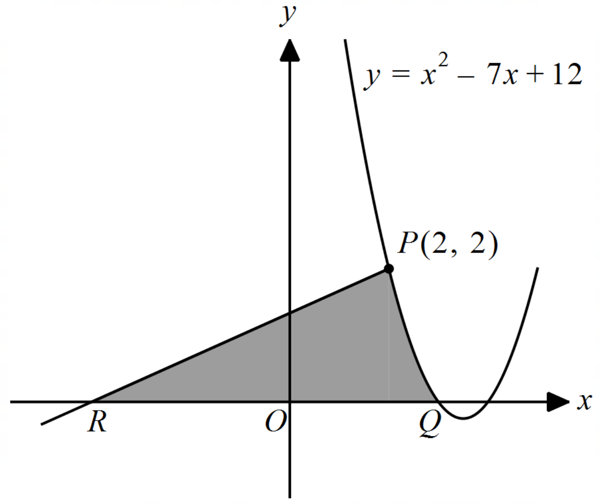
$$\therefore \frac{11x+12}{x^2(x+4)}=\frac{2}{x}+\frac{3}{x^2}-\frac{2}{x+4}$$

(ii) Hence find $\displaystyle\int \frac{11x+12}{x^2(x+4)}\ \mathrm{d}x.$. [2]

$$\int \frac{11x+12}{x^2(x+4)}\ \mathrm{d}x=\int \frac{2}{x}+\frac{3}{x^2}-\frac{2}{x+4}\ \mathrm{d}x$$

$$=2\ln x-\frac{3}{x}-2\ln(x+4)+c$$

12 The diagram shows part of the curve $y = x^2 - 7x + 12$, cutting the x-axis at Q.
The normal to the curve at $P(2,2)$ meets the x-axis at R.



Show that the area of the shaded region bounded by the x-axis, the line PR and the curve is $6\frac{5}{6}$ units^2. [8]

$$\frac{dy}{dx} = 2x - 7$$

when $x = 2$,

$$\frac{dy}{dx} = 2(2) - 7$$
$$= -3$$

Gradient of normal = $\frac{1}{3}$

$$y = \frac{1}{3}x + c$$

at (2, 2)

$$2 = \frac{2}{3} + c$$

$$c = \frac{4}{3}$$

Equation of normal: $y = \frac{1}{3}x + \frac{4}{3}$

when $y = 0$,

$$-\frac{1}{3}x = \frac{4}{3}$$
$$x = -4$$
$$R = (-4, 0)$$

when $y = 0$,

$$x^2 - 7x + 12 = 0$$

$$(x-3)(x-4) = 0$$

$$x = 3 \quad \text{or} \quad x = 4$$

$$Q = (3, 0)$$

$$\text{Area of shaded region} = \frac{1}{2}(6)(2) + \int_2^3 x^2 - 7x + 12 \, \mathrm{d}x$$

$$= 6 + \left[\frac{x^3}{3} - \frac{7}{2}x^2 + 12x \right]_2^3$$

$$= 6 + \left[(9 - \frac{63}{2} + 36) - (\frac{8}{3} - 14 + 24) \right]$$

$$= 6\frac{5}{6} \text{ units}^2 \quad \text{(shown)}$$

2025 PRELIM EXAM
CHUNG CHENG HIGH (YISHUN)
ADDITIONAL MATH (G3)
PAPER 2

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGNOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2\sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$\Delta = \frac{1}{2}bc\sin A$$

1 In 2020, there were an estimated 7 million insects of a particular species in a certain country. In 2021, the numbers were estimated to have fallen to 5 million. Scientists believe that the number of these insects, N million, can be modelled by the formula $N = 3 + ae^{-kt}$, where t is the time in years after 2020.

(a) Find the exact values of a and of k. [4]

(b) What is the approximate size of the population for large values of t? [1]

2 The equation of a curve is $y = px^2 - (p+2)x + 1$, where p is a constant.

(a) Given that $p > 0$, by completing the square, find the minimum value of y.

Express your answer in the form $\frac{a - p^2}{bp}$, where a and b are integers. [3]

(b) Given instead that $p < 0$, find the coordinates of the maximum point in terms of p. [2]

3 **(a)** Show that $\frac{\mathrm{d}}{\mathrm{d}x}(\cot x) = -\operatorname{cosec}^2 x$. [2]

(b) A curve has equation $y = \mathrm{e}^{-ax}\cot x$, where a is a positive constant. There is exactly one point in the interval $-\frac{1}{2}\pi < x < 0$ at which the tangent is parallel to the x-axis.
Find the value of a and state the exact x-coordinate of this point. [5]

4 It is given that $f(x) = 3x^3 + ax^2 - 7x + 3$, where a is a constant, has a factor of $x+1$.

(a) Find the value of a and factorise $f(x)$ completely. [4]

(b) Solve the equation $f(\cot^2\theta) = 0$ for $-90° < \theta < 90°$. [3]

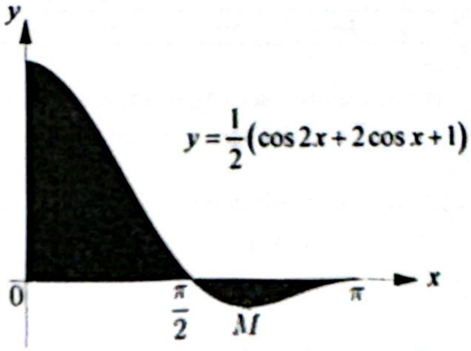
5 **(a)** Prove the identity $\sin 2x(a\cot x + b\tan x) = a + b + (a-b)\cos 2x$, where a and b are constants. [4]

(b) Hence solve the equation $\sin\frac{1}{2}\theta\left(3\cot\frac{1}{4}\theta + 5\tan\frac{1}{4}\theta\right) = 7$ for $0 \le \theta \le 4\pi$. [3]

6 (a) Solve the equation $\log_5(3y+1) - \frac{1}{\log_y 5} + \log_2 8 = 4$. [4]

(b) By expressing the equation $\ln\left(6e^x+1\right)+x=0$ as a quadratic equation in e^x, solve the equation $\ln\left(6e^x+1\right)+x=0$, giving values of x in logarithmic form. [4]

7 The diagram shows the curve $y=\frac{1}{2}(\cos 2x+2\cos x+1)$ for $0 \le x \le \pi$ radians.



The point M is the minimum point of the curve, where the x-coordinate of M lies in the interval $\frac{\pi}{2} < x < \pi$.

(a) Find the exact coordinates of M. [5]

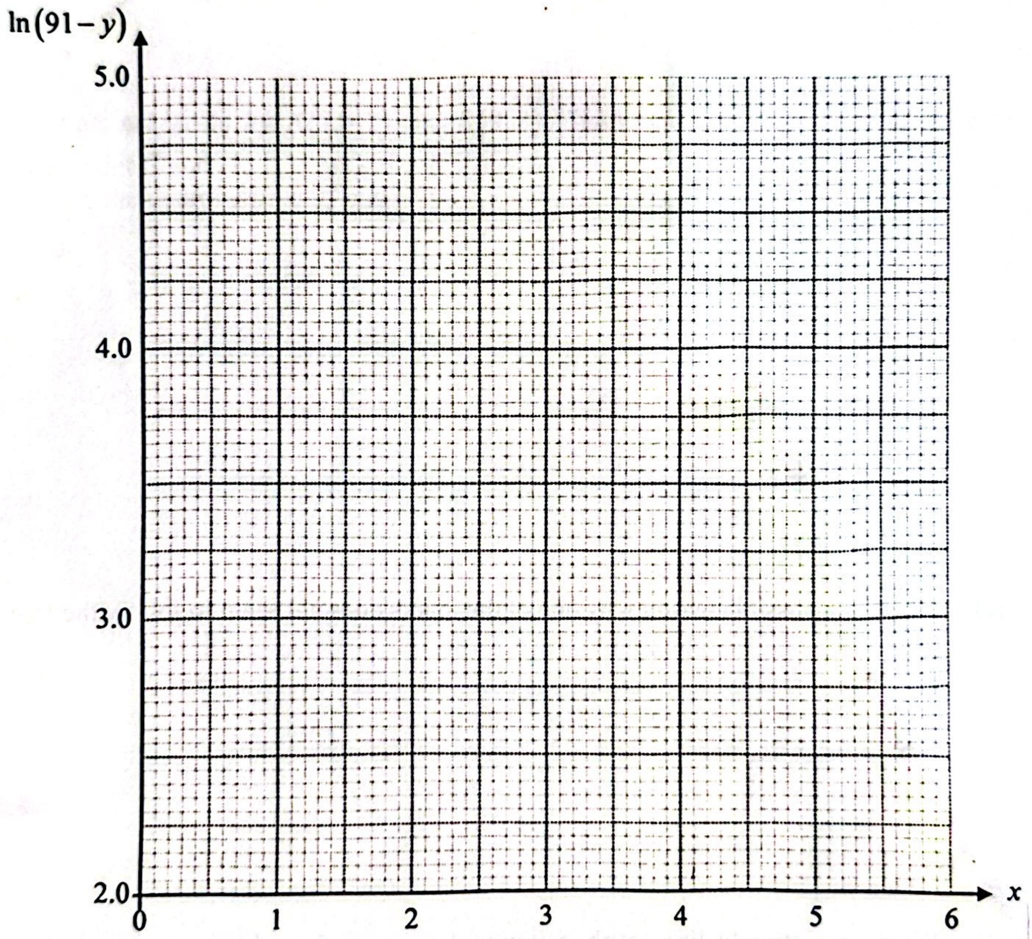
(b) Find the exact total area of the shaded regions bounded by the curve and the axes. **[4]**

8 The table shows Andy's marks for his Mathematics practice papers each week.

Week x	1	2	3	4	5	6
Marks y	45	59	68	74	71	82

He believed that these figures can be modelled by the formula $y = 91 - Ae^{kx}$, where A and k are constants, by excluding one datapoint that does not follow the trend.

(a) On the grid below, by ignoring the datapoint that does not follow the trend, plot $\ln(91-y)$ against x and draw a straight line graph. [2]



(b) Use the graph to estimate the value of each of the constants A and k. [5]

(c) Suggest a possible reason why one of the marks does not seem to follow the trend. [1]

(d) From your straight line graph, estimate the expected marks for the datapoint that was excluded. [2]

9 The equation of a circle is $x^2+y^2-8x-4y+15=0$.

(a) Find the radius and coordinates of the centre of the circle. [3]

Two points are given by $A(5,-2)$ and $B(9,2)$. The perpendicular bisector of AB cuts the circle at point C and D.

(b) Find the coordinates of C and of D. [6]

(c) Find the shortest distance from the origin O to the line segment CD, giving your answer in the form $k\sqrt{2}$, where k is a constant to be determined. **[3]**

10 A particle travels in a straight line so that its displacement, s m, from O at time t seconds, where $t \geq 0$, is modelled by $s = t^3 - \frac{9}{2}t^2 + 6t + 1$.

(a) Find the values of t for which the particle is instantaneously at rest. [3]

(b) The particle's acceleration is 15 m/s² at T seconds. Find the total distance travelled by the particle in the interval $t = 0$ to $t = T$. [4]

(c) Explain clearly why the answer found in part (b) **should not** be obtained by finding the value of s when $t = T$. [2]

11 (a) A curve $y = f(x)$ is such that $f'(x) = ax^2 - 6x + b$, where a and b are constants.

(i) Given that the curve is always increasing, what conditions must apply to a and b? [3]

It is now given that $a = 5$.

(ii) The curve intersects the y-axis at $(0, 4)$ and passes through the points $(-3, -80)$ and $(3, k)$. Find the value of k. [5]

This question is not related to part **(a)**.

(b) It is given that $y = \mathrm{g}(x)$ is a solution of the equation $\mathrm{e}^{-2x}\left(\frac{\mathrm{d}^2y}{\mathrm{d}x^2} - 2\frac{\mathrm{d}y}{\mathrm{d}x} + y\right) = -\mathrm{e}^k$, where k is a constant. The point $(1, 0)$ is a stationary point on the curve $y = \mathrm{g}(x)$.
Find the nature of this stationary point. **[3]**

中正中学 义顺

CHUNG CHENG HIGH SCHOOL (YISHUN)

2025 Preliminary Examination
Secondary Four Express / Five Normal Academic

CANDIDATE NAME		INDEX NUMBER	
FORM CLASS / SUBJECT GROUP	/	DATE	

ADDITIONAL MATHEMATICS — 4049/02

Paper 2

28 August 2025

2 hours 15 minutes

Candidates answer on the Question Paper.

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.
Up to 2 marks may be deducted for improper presentation.

The number of marks is given in brackets [ ] at the end of each question or part question.

Question Number	Marks Possible	Marks Obtained
1	5	
2	5	
3	7	
4	7	
5	7	
6	8	
7	9	
8	10	
9	12	
10	9	
11	11	
Presentation Deduction		– 1 / – 2
TOTAL	90	

This document consists of **18** printed pages.

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$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$\Delta = \frac{1}{2}bc\sin A$$

1 In 2020, there were an estimated 7 million insects of a particular species in a certain country. In 2021, the numbers were estimated to have fallen to 5 million. Scientists believe that the number of these insects, N million, can be modelled by the formula $N = 3 + ae^{-kt}$, where t is the time in years after 2020.

(a) Find the exact values of a and of k. [4]

(a)	When $t = 0, N = 7$. $7 = 3 + ae^{-k(0)}$ $a = 7 - 3$ $= 4$ When $t = 1, N = 5$. $5 = 3 + 4e^{-k(1)}$ $2 = 4e^{-k}$ $e^{-k} = \frac{2}{4}$ $-k = \ln\frac{1}{2}$ $k = \ln 2$	

(b) What is the approximate size of the population for large values of t? [1]

(b)	$N = 3 + 4e^{-(\ln 2)t}$ When t is large, $e^{-(\ln 2)t}$ becomes close to zero. Thus, the approximate size is 3 million.	

2 The equation of a curve is $y = px^2 - (p+2)x + 1$, where p is a constant.

(a) Given that $p > 0$, by completing the square, find the minimum value of y.

Express your answer in the form $\frac{a - p^2}{bp}$, where a and b are integers. [3]

(a)	$y = px^2 - (p+2)x + 1$ $= p\left(x^2 - \frac{p+2}{p}x\right) + 1$ $= p\left[x^2 - \frac{p+2}{p}x + \left(\frac{p+2}{2p}\right)^2 - \left(\frac{p+2}{2p}\right)^2\right] + 1$ $= p\left[\left(x - \frac{p+2}{2p}\right)^2 - \left(\frac{p+2}{2p}\right)^2\right] + 1$ $= p\left(x - \frac{p+2}{2p}\right)^2 + 1 - \frac{p^2+4p+4}{4p^2} \times p$ $= p\left(x - \frac{p+2}{2p}\right)^2 + \frac{4p - p^2 - 4p - 4}{4p}$ $= p\left(x - \frac{p+2}{2p}\right)^2 + \frac{-p^2-4}{4p}$ Hence, the minimum value of y is $\frac{-p^2-4}{4p}$.	

(b) Given instead that $p < 0$, find the coordinates of the maximum point in terms of p. [2]

(b)	$\left(\frac{p+2}{2p}, \frac{-p^2-4}{4p}\right)$	

3 **(a)** Show that $\dfrac{\mathrm{d}}{\mathrm{d}x}(\cot x) = -\operatorname{cosec}^2 x$. [2]

(a)	$\dfrac{\mathrm{d}}{\mathrm{d}x}(\cot x)$ $= \dfrac{\mathrm{d}}{\mathrm{d}x}\left(\dfrac{\cos x}{\sin x}\right)$ $= \dfrac{\sin x(-\sin x) - \cos x(\cos x)}{\sin^2 x}$ $= \dfrac{-\sin^2 x - \cos^2 x}{\sin^2 x}$ $= \dfrac{-1}{\sin^2 x}$ or $-1-\cot^2 x$ $= -\operatorname{cosec}^2 x$ $\dfrac{\mathrm{d}}{\mathrm{d}x}(\cot x)$ $= \dfrac{\mathrm{d}}{\mathrm{d}x}\left(\dfrac{1}{\tan x}\right) = \dfrac{\mathrm{d}}{\mathrm{d}x}(\tan x)^{-1}$ $= -1(\tan x)^{-2}(\sec^2 x)$ $= -\dfrac{1}{\tan^2 x \cos^2 x}$ $= \dfrac{-1}{\sin^2 x}$ $= -\operatorname{cosec}^2 x$	

(b) A curve has equation $y = \mathrm{e}^{-ax}\cot x$, where a is a positive constant. There is exactly one point in the interval $-\dfrac{1}{2}\pi < x < 0$ at which the tangent is parallel to the x-axis. Find the value of a and state the exact x-coordinate of this point. [5]

(b)	$\dfrac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{e}^{-ax}\left(-\operatorname{cosec}^2 x\right) - a\mathrm{e}^{-ax}\cot x$ $= -\mathrm{e}^{-ax}\left(\operatorname{cosec}^2 x + a\cot x\right)$ The tangent is parallel to x-axis means $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 0$. $-\mathrm{e}^{-ax}\left(\operatorname{cosec}^2 x + a\cot x\right) = 0$ Since $\mathrm{e}^{-ax} > 0$ for all values of x, $-\left(\operatorname{cosec}^2 x + a\cot x\right) = 0$ $\operatorname{cosec}^2 x + a\cot x = 0$ $\cot^2 x + 1 + a\cot x = 0$ Since there is only 1 point, the above equation only has 1 solution, i.e. discriminant = 0. $a^2 - 4(1)(1) = 0$ $a = \pm 2$ $a = 2$ $(a > 0)$ $\cot^2 x + 2\cot x + 1 = 0$ $(\cot x + 1)^2 = 0$ $\cot x = -1$ $\tan x = -1$ $x = -\dfrac{\pi}{4}$	

4 It is given that $\mathrm{f}(x)=3x^3+ax^2-7x+3$, where a is a constant, has a factor of $x+1$.

(a) Find the value of a and factorise $\mathrm{f}(x)$ completely. [4]

(a)	$\mathrm{f}(-1)=3(-1)^3+a(-1)^2-7(-1)+3$ $0=-3+a+7+3$ $a=-7$ By long division, $\mathrm{f}(x)=3x^3-7x^2-7x+3$ $=(x+1)(3x^2-10x+3)$ $=(x+1)(3x-1)(x-3)$	

(b) Solve the equation $\mathrm{f}(\cot^2\theta)=0$ for $-90°<\theta<90°$. [3]

(b)	$\cot^2\theta=-1$ (rej) or $\cot^2\theta=\frac{1}{3}$ or $\cot^2\theta=3$ $\cot\theta=\pm\frac{1}{\sqrt{3}}$, $\theta=\pm60°$ $\cot\theta=\pm\sqrt{3}$, $\theta=\pm30°$	

5 **(a)** Prove the identity $\sin 2x(a\cot x + b\tan x) = a + b + (a-b)\cos 2x$, where a and b are constants. [4]

(a)	$\sin 2x(a\cot x + b\tan x) = 2\sin x\cos x\left(\frac{a\cos x}{\sin x} + \frac{b\sin x}{\cos x}\right)$ $= 2a\cos^2 x + 2b\sin^2 x$ $= a(\cos 2x + 1) + b(1-\cos 2x)$ $= a\cos 2x + a + b - b\cos 2x$ $= a + b + (a-b)\cos 2x$	

(b) Hence solve the equation $\sin\frac{1}{2}\theta\left(3\cot\frac{1}{4}\theta + 5\tan\frac{1}{4}\theta\right) = 7$ for $0 \le \theta \le 4\pi$. [3]

(b)	$3 + 5 + (3-5)\cos 2\left(\frac{1}{4}\theta\right) = 7$ $-2\cos\left(\frac{1}{2}\theta\right) = -1$ $\cos\left(\frac{1}{2}\theta\right) = \frac{1}{2}$ basic angle $= \cos^{-1}\frac{1}{2} = \frac{\pi}{3}$ $\frac{1}{2}\theta$ is in 1st or 4th quadrant. $\frac{1}{2}\theta = \frac{\pi}{3}$ or $2\pi - \frac{\pi}{3}$ $\theta = \frac{2\pi}{3}$ or $\frac{10\pi}{3}$	

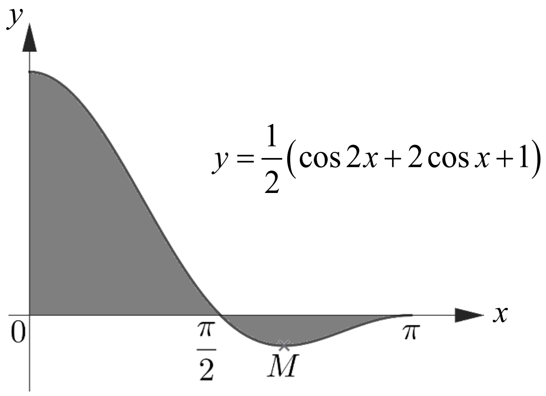
6 **(a)** Solve the equation $\log_5(3y+1)-\dfrac{1}{\log_y 5}+\log_2 8=4$. [4]

(a)	$\log_5(3y+1)-\dfrac{1}{\log_y 5}+\log_2 8=4$ $\log_5(3y+1)-\log_5 y=4-\log_2 8$ $\log_5\left(\dfrac{3y+1}{y}\right)=4-\log_2 2^3$ $\log_5\left(3+\dfrac{1}{y}\right)=4-3$ $3+\dfrac{1}{y}=5^1$ $\dfrac{1}{y}=2$ $y=\dfrac{1}{2}$	

(b) By expressing the equation $\ln(6e^x+1)+x=0$ as a quadratic equation in e^x, solve the equation $\ln(6e^x+1)+x=0$, giving values of x in logarithmic form. [4]

(b)	$\ln(6e^x+1)+x=0$ $\ln(6e^x+1)=-x$ $6e^x+1=e^{-x}$ $6(e^x)^2+(e^x)-1=0$ $(3e^x-1)(2e^x+1)=0$ $e^x=\dfrac{1}{3}$ or $e^x=-\dfrac{1}{2}$ (rej) $x=\ln\dfrac{1}{3}$ or $-\ln 3$	

7 The diagram shows the curve $y=\frac{1}{2}(\cos 2x+2\cos x+1)$ for $0 \le x \le \pi$ radians.



The point M is the minimum point of the curve, where the x-coordinate of M lies in the interval $\frac{\pi}{2} < x < \pi$.

(a) Find the exact coordinates of M. [5]

(a)	$y=\frac{1}{2}(\cos 2x+2\cos x+1)$ $\frac{dy}{dx}=\frac{1}{2}(-2\sin 2x-2\sin x)$ $=-\sin 2x-\sin x$ At min pt, $\frac{dy}{dx}=0$. $-\sin 2x-\sin x=0$ $-2\sin x\cos x-\sin x=0$ $-\sin x(2\cos x+1)=0$ $-\sin x=0$ or $\cos x=-\frac{1}{2}$ $x=0$ or $\frac{\pi}{2}$ (rej) $\quad$ $x=\pi-\frac{\pi}{3}$ or $\pi+\frac{\pi}{3}$ (rej) $y=-\frac{1}{4}$ $\therefore M\left(\frac{2\pi}{3},-\frac{1}{4}\right)$	

(b) Find the exact total area of the shaded regions bounded by the curve and the axes. [4]

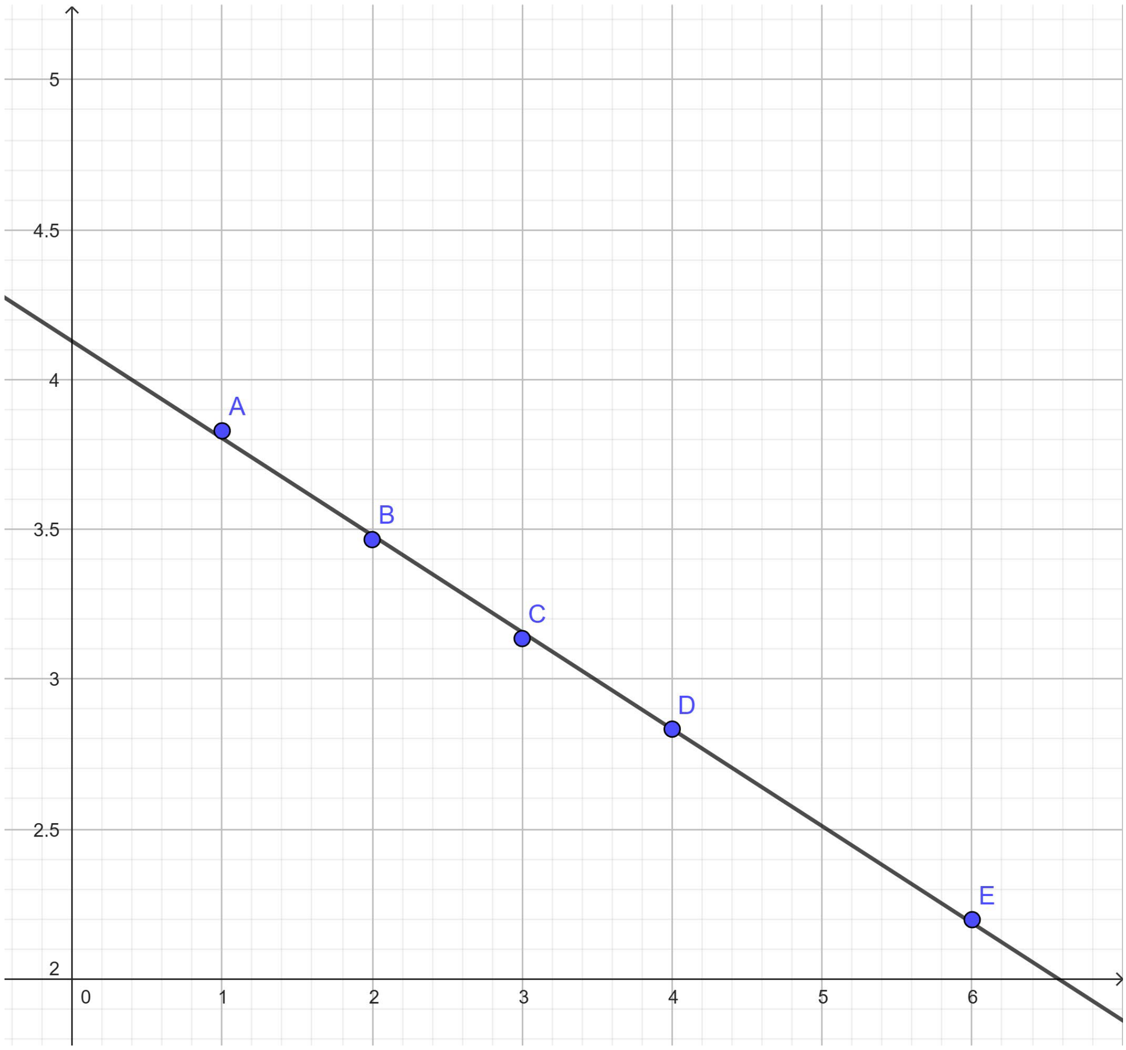
(b)	area $= \int_0^{\frac{\pi}{2}} \frac{1}{2}\left(\cos 2x + 2\cos x + 1\right)\ \mathrm{d}x - \int_{\frac{\pi}{2}}^{\pi} \frac{1}{2}\left(\cos 2x + 2\cos x + 1\right)\ \mathrm{d}x$ $= \frac{1}{2}\left[\frac{\sin 2x}{2} + 2\sin x + x\right]_0^{\frac{\pi}{2}} - \frac{1}{2}\left[\frac{\sin 2x}{2} + 2\sin x + x\right]_{\frac{\pi}{2}}^{\pi}$ $= \left[\left(0+1+\frac{\pi}{4}\right) - (0+0+0)\right] - \left[\left(0+0+\frac{\pi}{2}\right) - \left(0+1+\frac{\pi}{4}\right)\right]$ $= 2$ units^2	

8 The table shows Andy's marks for his Mathematics practice papers each week.

Week x	1	2	3	4	5	6
Marks y	45	59	68	74	71	82

He believed that these figures can be modelled by the formula $y = 91 - Ae^{kx}$, where A and k are constants, by excluding one datapoint that does not follow the trend.

(a) On the grid below, by ignoring the datapoint that does not follow the trend, plot $\ln(91 - y)$ against x and draw a straight line graph. [2]



(b) Use the graph to estimate the value of each of the constants A and k. [5]

(b)	Transforming the equation: $$y = 91 - Ae^{kx}$$ $$91 - y = Ae^{kx}$$ $$\ln(91 - y) = \ln A + \ln e^{kx}$$ $$\ln(91 - y) = \ln A + kx$$ Using 2 points on the line (1, 3.8) and (3.5, 3), $$\text{gradient} = \frac{3.8 - 3}{1 - 3.5} = -0.32$$ Reading off the Y-intercept, we have 4.15 The equation of line is $$\ln(91 - y) = -0.32x + 4.15$$ By comparing, $\ln A = 4.15$ $\quad\quad k = -0.32$ $A = 63.4$ (3 s.f.)	

(c) Suggest a possible reason why one of the marks does not seem to follow the trend. [1]

(c)	Any reason within the context - The lower mark on week 5 was likely to be due to a harder paper. - Andy might have been ill.	

(d) From your straight line graph, estimate the expected marks for the datapoint that was excluded. [2]

(d)	The datapoint is (5, 71) Based on the straight line graph, the reading should be 2.5. The expected marks is $$\ln(91 - y) = 2.5$$ $$y = 79$$	

9 The equation of a circle is $x^2 + y^2 - 8x - 4y + 15 = 0$.

(a) Find the radius and coordinates of the centre of the circle. [3]

(a)	$x^2 + y^2 - 8x - 4y + 15 = 0$ $x^2 - 8x + 4^2 + y^2 - 4y + 2^2 = 4^2 + 2^2 - 15$ $(x-4)^2 + (y-2)^2 = 5$ Radius is $\sqrt{5}$ units Centre (4, 2)	

Two points are given by $A(5, -2)$ and $B(9, 2)$. The perpendicular bisector of AB cuts the circle at point C and D.

(b) Find the coordinates of C and of D. [6]

(b)	$m_{AB} = \frac{-2-2}{5-9}$ $= 1$ Midpoint of $AB = \left(\frac{5+9}{2}, \frac{-2+2}{2}\right)$ $= (7, 0)$ Equation of perpendicular bisector is $y - 0 = -1(x-7)$ $y = -x + 7$ Substituting equation of line into equation of circle: $(x-4)^2 + (-x+7-2)^2 = 5$ $x^2 - 8x + 16 + x^2 - 10x + 25 = 5$ $2x^2 - 18x + 36 = 0$ $x^2 - 9x + 18 = 0$ $(x-3)(x-6) = 0$ $x = 3$ or $x = 6$ $y = 4$ $\quad$ $y = 1$ The coordinates are (3, 4) and (6, 1).	

(c) Find the shortest distance from the origin O to the line segment CD, giving your answer in the form $k\sqrt{2}$, where k is a constant to be determined. [3]

(c)	Area of $OCD = \frac{1}{2}\begin{vmatrix} 0 & 6 & 3 & 0 \\ 0 & 1 & 4 & 0 \end{vmatrix}$ $= \frac{1}{2}\left[(0+24+0)-(0+3+0)\right]$ $= 10.5$ Length of $CD = \sqrt{(3-6)^2+(4-1)^2}$ $= \sqrt{18}$ $= 3\sqrt{2}$ shortest dist $= \dfrac{10.5}{\frac{1}{2}\times 3\sqrt{2}}$ $= \dfrac{7}{\sqrt{2}}$ $= \frac{7}{2}\sqrt{2}$ *Alternatively,* The perpendicular line from O to CD has gradient 1, since the gradient of CD is –1. Thus the perpendicular line has equation $y = x$, since the y-intercept is 0. Intersecting the 2 lines $y = x$ and $y = -x+7$, we have $x = -x+7$ $2x = 7$ $x = \frac{7}{2}$ $y = \frac{7}{2}$ The point on CD shortest distance from O is $\left(\frac{7}{2}, \frac{7}{2}\right)$. Thus, the distance is $\sqrt{\left(\frac{7}{2}-0\right)^2+\left(\frac{7}{2}-0\right)^2} = \sqrt{2\left(\frac{7}{2}\right)^2}$ $= \frac{7}{2}\sqrt{2}$	

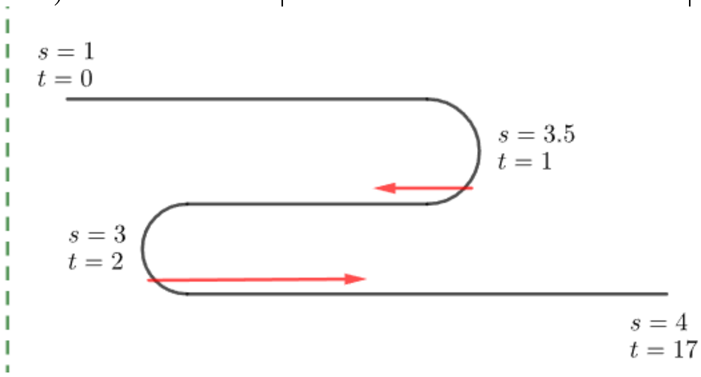
10 A particle travels in a straight line so that its displacement, s m, from O at time t seconds, where $t \geq 0$, is modelled by $s = t^3 - \frac{9}{2}t^2 + 6t + 1$.

(a) Find the values of t for which the particle is instantaneously at rest. [3]

(a)	$v = \frac{ds}{dt} = 3t^2 - 9t + 6$ When particle is at instantaneously rest, $v = 0$. $3t^2 - 9t + 6 = 0$ $t^2 - 3t + 2 = 0$ $(t-1)(t-2) = 0$ $t = 1$ or $t = 2$	

(b) The particle's acceleration is 15 m/s^2 at T seconds. Find the total distance travelled by the particle in the interval $t = 0$ to $t = T$. [4]

(b)	$a = \frac{dv}{dt} = 6t - 9$ When $a = 15$, $6T - 9 = 15$ $T = 4$ When $t = 0$, $s = 1$ When $t = 1$, $s = 3.5$ When $t = 2$, $s = 3$ When $t = 4$, $s = 17$ Total distance $= (3.5-1) + (3.5-3) + (17-3)$ $= 17$	



(c) Explain clearly why the answer found in part **(b) should not** be obtained by finding the value of s when $t = T$. [2]

(c)	Even though the numerical value is the same, the particle turned twice (at least once), thus will need to account for the additional distance. Also, evaluating s at $t = T$ only gave the distance of the particle from O, which is not what is required.	

11 **(a)** A curve $y = \mathrm{f}(x)$ is such that $\mathrm{f}'(x) = ax^2 - 6x + b$, where a and b are constants.

(i) Given that the curve is always increasing, what conditions must apply to a and b? [3]

(a)	For the curve to be always increasing, i.e. $\mathrm{f}'(x) > 0$, that is $ax^2 - 6x + b > 0$ This means that there are no roots, i.e. discriminant < 0. $(-6)^2 - 4(a)(b) < 0$ $36 - 4ab < 0$ $36 < 4ab$ $ab > 9$ and $a > 0$	

It is now given that $a = 5$.

(ii) The curve intersects the y-axis at $(0, 4)$ and passes through the points $(-3, -80)$ and $(3, k)$. Find the value of k. [5]

(b)	$\frac{\mathrm{d}y}{\mathrm{d}x} = 5x^2 - 6x + b$ $y = \frac{5}{3}x^3 - 3x^2 + bx + 4$ When $x = -3,\ y = 80$. $80 = \frac{5}{3}(-3)^3 - 3(-3)^2 + b(-3) + 4$ $-80 = -45 - 27 - 3b + 4$ $3b = 12$ $b = 4$ When $x = 3,\ y = k$. $k = \frac{5}{3}(3)^3 - 3(3)^2 + 4(3) + 4$ $= 45 - 27 + 12 + 4$ $= 34$	

This question is not related to part **(a)**.

(b) It is given that $y = \mathrm{g}(x)$ is a solution of the equation $\mathrm{e}^{-2x}\left(\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} - 2\frac{\mathrm{d}y}{\mathrm{d}x} + y\right) = -\mathrm{e}^k$, where k is a constant. The point $(1, 0)$ is a stationary point on the curve $y = \mathrm{g}(x)$.
Find the nature of this stationary point. [3]

(b)	Since it is a stationary point, $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$. Together with (1, 0), we can substitute $x = 1,\ y = 0,\ \frac{\mathrm{d}y}{\mathrm{d}x} = 0$. The equation is now reduced to $\mathrm{e}^{-2(1)}\left(\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} - 2(0) + 0\right) = -\mathrm{e}^k$ $\mathrm{e}^{-2}\left(\frac{\mathrm{d}^2 y}{\mathrm{d}x^2}\right) = -\mathrm{e}^k$ $\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} = -\mathrm{e}^{2+k}$ Since exponential functions are always positive, we have $-\mathrm{e}^{2+k}$ to be negative. Since $\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} < 0$, by 2nd derivative test, (1, 0) is a maximum point.	

Bedok View Secondary School
Department of Mathematics — Mark Scheme

Year	2025	Level & Stream	4E5N
Type of Assessment	Preliminary Exam	Subject & Paper	Additional Math P1

Qns	Working
1	$4x^2-6xy-6y=4$ ----(1) $y-x=4$ ------(2) Sub (2) into (1): $4x^2-6x(4+x)-6(4+x)=4$ $4x^2-24x-6x^2-24-6x=4$ $2x^2+30x+28=0$ $x^2+15x+14=0$ $(x+1)(x+14)=0$ $x=-14$ or $x=-1$ $y=-10$ or $y=3$
2	$9\sin x=-2\sec x$ $9\sin x=-\frac{2}{\cos x}$ $\sin x\cos x=-\frac{2}{9}$ $2\sin x\cos x=-\frac{4}{9}$ $\sin 2x=-\frac{4}{9}$ Basic angle = $\sin^{-1}\left(\frac{4}{9}\right)$ = 0.46055 $2x=-0.46055,\ -(\pi-0.46055)$ $=-0.46055$ or -2.68104 $x=-0.230$ or -1.34

Qns	Working
3a	Principal range of $\cos^{-1}x$ is $0 \le \cos^{-1}x \le \pi$, hence principal value of $\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)$ is $\frac{3\pi}{4}$ instead of $-\frac{\pi}{4}$.
3b	L.H.S: $\cot A + \tan A = \frac{1}{\tan A} + \tan A$ $= \frac{\cos A}{\sin A} + \frac{\sin A}{\cos A}$ $= \frac{\cos^2 A + \sin^2 A}{\sin A \cos A}$ $= \frac{1}{\sin A \cos A}$ $= \sec A \operatorname{cosec} A = \text{RHS (shown)}$ **OR** RHS: $\sec A \operatorname{cosec} A = \frac{1}{\cos A} \times \frac{1}{\sin A}$ $\frac{1}{\cos A \sin A}$ $= \frac{\sin^2 A + \cos^2 A}{\cos A \sin A}$ $= = \frac{\sin^2 A}{\cos A \sin A} + \frac{\cos^2 A}{\cos A \sin A}$ $= \frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}$ $= \tan A + \cot A = \text{LHS (shown)}$

Qns	Working
4	By long division, $\frac{2x^3+4}{x(x+2)^2}=\frac{2x^3+4}{x^3+4x^2+4x}$ $=2+\frac{-8x^2-8x+4}{x(x+2)^2}$ $\frac{-8x^2-8x+4}{x(x+2)^2}=\frac{A}{x}+\frac{B}{x+2}+\frac{C}{(x+2)^2}$ $-8x^2-8x+4=A(x+2)^2+Bx(x+2)+Cx$ Let $x=0$: $4=4A$ $A=1$ Let $x=-2$: $-12=-2C$ $C=6$ Let $x=1$: $-8-8+4=9+3B+6$ $B=-9$ $\therefore \frac{2x^3+4}{x(x+2)^2}=2+\frac{1}{x}-\frac{9}{x+2}+\frac{6}{(x+2)^2}$
5a	$\mathrm{f}'(x)=\frac{(x^2+3)-2x(x+a)}{(x^2+3)^2}$ $=\frac{-x^2-2ax+3}{(x^2+3)^2}$
5b	$\frac{-x^2-2ax+3}{(x^2+3)^2}>0,\ \text{for } b<x<1$ Since $(x^2+3)^2>0$, $-x^2-2ax+3>0$ $x^2+2ax-3<0$ Given that $b<x<1$ $(x-b)(x-1)<0$ $x^2-bx-x+b<0$ By comparing: $b=-3$ $2a=-b-1$ $2a=3-1$ $a=1$

Qns	Working
6	$y=(k-6)x^2-8x+k$ Discriminant $=(-8)^2-4(k-6)(k)$ $=64-4k^2+24k$ For curve that does not intersect the x-axis, Discriminant < 0 $k^2-6k-16>0$ $(k+2)(k-8)>0$ $k<-2$ or $k>8$ The curve has a minimum point: $k-6>0$ $k>6$ $\therefore k>8$
7a 7b	$\frac{\mathrm{d}}{\mathrm{d}x}\left[(x-1)\mathrm{e}^{-x}\right]=\mathrm{e}^{-x}-(x-1)\mathrm{e}^{-x}$ $=2\mathrm{e}^{-x}-x\mathrm{e}^{-x}$ $\int 2\mathrm{e}^{-x}-x\mathrm{e}^{-x}\mathrm{d}x=(x-1)\mathrm{e}^{-x}+c$ $\int 2\mathrm{e}^{-x}\mathrm{d}x-\int x\mathrm{e}^{-x}\mathrm{d}x=(x-1)\mathrm{e}^{-x}+c$ $-\int x\mathrm{e}^{-x}\mathrm{d}x=-\int 2\mathrm{e}^{-x}\mathrm{d}x+(x-1)\mathrm{e}^{-x}+c$ $\int x\mathrm{e}^{-x}\mathrm{d}x=-2\mathrm{e}^{-x}-(x-1)\mathrm{e}^{-x}+c$ $=-2\mathrm{e}^{-x}-x\mathrm{e}^{-x}+\mathrm{e}^{-x}+c$ $=-\mathrm{e}^{-x}-x\mathrm{e}^{-x}+c$

Qns	Working
8a	$\frac{dr}{dt} = -\frac{2}{(t+1)^2}$ $r = -\int \frac{2}{(t+1)^2} dt$ $= -\int 2(t+1)^{-2} dt$ $= 2(t+1)^{-1} + c$ $r = \frac{2}{t+1} + c$ At $r = 4$ and $t = 0$, $4 = 2 + c$ $c = 2$ $r = \frac{2}{t+1} + 2$
8b	$A = \pi r^2$ $\frac{dA}{dr} = 2\pi r$ At $r = 2.6$, $\frac{dA}{dr} = 5.2\pi$, $2.6 = \frac{2}{t+1} + 2$ $0.6 = \frac{2}{t+1}$ $t = \frac{7}{3}$ $\frac{dr}{dt} = -\frac{2}{\left(\frac{7}{3}+1\right)^2} = -0.18$ $\frac{dA}{dt} = 5.2\pi \times -0.18$ $\frac{dA}{dt} = -0.936\pi \text{ cm}^2/\text{min}$

Qns	Working
9a	$y = kx + 6$ ---- (1) $2x^2 - xy = 3$ -----(2) Sub (1) into (2): $2x^2 - x(kx+6) = 3$ $2x^2 - kx^2 - 6x - 3 = 0$ $(2-k)x^2 - 6x - 3 = 0$ Discriminant $= (-6)^2 - 4(2-k)(-3)$ $= 60 - 12k$ For tangent, $60 - 12k = 0$ $k = 5$
9b	$\left(4 - 2\sqrt{2}\right)x + \sqrt{2} = 3x\sqrt{2} - 1$ $4x - 2x\sqrt{2} + \sqrt{2} = 3x\sqrt{2} - 1$ $\sqrt{2} + 1 = 3x\sqrt{2} + 2x\sqrt{2} - 4x$ $\sqrt{2} + 1 = x\left(5\sqrt{2} - 4\right)$ $x = \frac{\sqrt{2}+1}{5\sqrt{2}-4} \times \frac{5\sqrt{2}+4}{5\sqrt{2}+4}$ $= \frac{5(2) + 4\sqrt{2} + 5\sqrt{2} + 4}{\left(5\sqrt{2}\right)^2 - 4^2}$ $= \frac{9\sqrt{2} + 14}{50 - 16}$ $= \frac{9}{34}\sqrt{2} + \frac{7}{17}$

Qns	Working
10a	Area of $\Delta PQR = \frac{1}{2}\begin{vmatrix} 1 & 2 & 0 & 1 \\ 0 & 3 & 2 & 0 \end{vmatrix}$ $= \frac{1}{2}[3+4-2]$ $= 2.5 \text{ units}^2$
10b	$m_{PR} = \frac{3-0}{2-1}$ $= 3$ $m_{QS} = -\frac{1}{3}$ Equation of QS: $y-2 = -\frac{1}{3}(x-0)$ $y = -\frac{1}{3}x + 2 ---(1)$ $x - y = 5 ---(2)$ Sub (1) into (2): $x + \frac{1}{3}x - 2 = 5$ $x = \frac{21}{4}$ $y = \frac{1}{4}$ coordinates of $S = \left(\frac{21}{4}, \frac{1}{4}\right)$

Qns	Working
11a	$2^x + 4^{x-1} = 24$ $2^x + \left(4^x\right)\left(4^{-1}\right) = 24$ $2^x + \frac{2^{2x}}{4} = 24$ Let $y = 2^x$, $y + \frac{y^2}{4} = 24$ $4y + y^2 = 96$ $y^2 + 4y - 96 = 0$ $\left(y-8\right)\left(y+12\right) = 0$ $y = 8$ or $y = -12$ $2^x = 8$ or $2^x = -12$ $\left(\text{Rejected since } 2^x > 0\right)$ $2^x = 2^3$ $x = 3$
11b	$\log_{\sqrt{3}}(x-1) = \log_3(x+5)$ $\frac{\log_3(x-1)}{\log_3\sqrt{3}} = \log_3(x+5)$ $\frac{\log_3(x-1)}{\log_3 3^{\frac{1}{2}}} = \log_3(x+5)$ $2\log_3(x-1) = \log_3(x+5)$ $\log_3(x-1)^2 = \log_3(x+5)$ $(x-1)^2 = x+5$ $x^2 - 2x + 1 - x - 5 = 0$ $x^2 - 3x - 4 = 0$ $(x+1)(x-4) = 0$ $x + 1 = 0$ $x = -1$ [N.A. as $\log_{\sqrt{3}}(x-1)$ will be undefined] or $x - 4 = 0$ $x = 4$

Qns	Working
12a	$4x-4y=3$ $y=x-\dfrac{3}{4}$ m of normal $=1$ m of tangent $=-1$ At $x=\dfrac{1}{2}$ and $\dfrac{dy}{dx}=-1$, $6\left(\dfrac{1}{2}\right)-\dfrac{1}{k\left(\dfrac{1}{2}\right)^3}=-1$ $3-\dfrac{8}{k}=-1$ $k=2$ (shown)
12b	At stat points, $6x-\dfrac{1}{2x^3}=0$ $6x=\dfrac{1}{2x^3}$ $x^4=\dfrac{1}{12}$ $x=\pm\sqrt[4]{\dfrac{1}{12}}$ $x=-0.537$ or 0.537
12c	$\dfrac{d^2y}{dx^2}=6+\dfrac{3}{2x^4}$ $6>0$ $\dfrac{3}{2x^4}>0$ $\therefore \dfrac{d^2y}{dx^2}=6+\dfrac{3}{2x^4}>0$ Since $\dfrac{d^2y}{dx^2}=6+\dfrac{3}{2x^4}>0$ the gradient has no turning point.

Qns	Working
13a	$C = 1.2n^2 - 14.4n + 53.7$ At $n = 0$, $C = \$53.7$ thousands
13b	$C = 1.2n^2 - 14.4n + 53.7$ $= 1.2\left[n^2 - 12n + 44.75\right]$ $= 1.2\left[(n-6)^2 - 6^2 + 44.75\right]$ $= 1.2(n-6)^2 + 10.5$
13c	For 600 pairs of running shoes produced, the minimum cost of 10.5 thousand dollars will be incurred.
13d	$1.2(n-6)^2 + 10.5 = 50$ $1.2(n-6)^2 = 39.5$ $(n-6)^2 = \frac{39.5}{1.2}$ $n - 6 = \pm 5.7373$ $n = 11.7373$ or 0.2627 Maximum number of pairs of running shoes for which the cost is at most 50 thousand dollars = 11.7373 = 11.7 hundreds

Qns	Working
14a	$m_{AB}=1$ Equation of AB: $y+2=x+4$ $y=x+2----(1)$ $y=-x+4----(2)$ $(1)=(2): x+2=-x+4$ $2x=2$ $x=1$ $y=3$ Coordinates of A = (1, 3)
14b	Radius $=\sqrt{(1+4)^2+(3+2)^2}$ $=\sqrt{50}$ units Equation of circle: $(x+4)^2+(y+2)^2=50$
14c	Let the coordinates of the other end of the diameter passes through $A = C(x, y)$. Since AC is the diameter, B is the midpoint of AC. $\left(\frac{x+1}{2},\ \frac{y+3}{2}\right)=(-4,\ -2)$ $\frac{x+1}{2}=-4,\quad \frac{y+3}{2}=-2$ $x=-9,\ y=-7$ Equation of another tangent which is parallel to $y=-x+4$: $y+7=-1(x+9)$ $y=-x-16$

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \cdots + \binom{n}{r} a^{n-r} b^r + \cdots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!\,(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 Solve the simultaneous equations

[4]

[Turn over

$$y - x = 4,$$
$$4x^2 - 6xy - 6y = 4.$$

2 Solve the equation $9\sin x = -2\sec x$ for $-\pi \le x \le 0$. [4]

For Examiner's Use

3 **(a)** Explain why the principal value of $\cos^{-1}\left(-\dfrac{\sqrt{2}}{2}\right)$ cannot be $-\dfrac{\pi}{4}$. [1]

[Turn over

For Examiner's Use

(b) Prove that $\cot A + \tan A = \sec A \operatorname{cosec} A$. [4]

For Examiner's Use

4 Express $\dfrac{2x^3+4}{x(x+2)^2}$ in partial fractions. [5]

[Turn over

For Examiner's Use

5 The function f is given by $f(x) = \dfrac{x + a}{x^2 + 3}$.

(a) Find $f'(x)$. [2]

It is given that f increases for $b < x < 1$.

(b) Find the values of a and b. [4]

For Examiner's Use

6 Find the set of values of the constant k for which the curve $y=(k-6)x^2-8x+k$ does not intersect the x-axis and has a minimum point. [6]

[Turn over

7 **(a)** Find $\frac{\mathrm{d}}{\mathrm{d}x}\left[\left(x-1\right)\mathrm{e}^{-x}\right]$. [2]

(b) Hence find $\int x\mathrm{e}^{-x}\,\mathrm{d}x$. [4]

For Examiner's Use

8 The radius of a circle, r cm, decreases at a rate of $\dfrac{2}{(t+1)^2}$ cm per minute.

(a) Given that the initial radius is 4 cm, find an expression for r in terms of t. [3]

(b) Find the rate of change of the area of the circle when the radius is 2.6 cm. [3]

[Turn over

For Examiner's Use

9 **(a)** Find the value of the constant k such that the line $y = kx + 6$ is a tangent to the curve $2x^2 - xy = 3$. [3]

(b) Solve $\left(4 - 2\sqrt{2}\right)x + \sqrt{2} = 3x\sqrt{2} - 1$, giving your answer in the form $a\sqrt{2} + b$, where a and b are rational numbers. [4]

For Examiner's Use

10 An isosceles triangle PQR has vertices $P(1, 0)$, $Q(0, 2)$ and $R(2, 3)$.
$PQ = QR$.

(a) Find the area of the triangle PQR. [2]

(b) If S is a point such that $PQRS$ is a kite and it also lies on the line $x - y = 5$, find the coordinates of S. [5]

11 **(a)** Solve the equation $2^{x} + 4^{x-1} = 24$. [4]

For Examiner's Use

(b) Solve the equation $\log_{\sqrt{3}}(x-1) = \log_3(x+5)$. [4]

For Examiner's Use

12 The gradient at any point on a curve y is given by $6x - \frac{1}{kx^3}$. The line $4x - 4y = 3$ is a normal to the curve at the point where $x = \frac{1}{2}$.

(a) Show that $k = 2$. [2]

(b) Hence find the x-coordinates of the stationary points. [2]

(c) Find $\frac{d^2y}{dx^2}$ and explain whether the gradient $6x - \frac{1}{kx^3}$ has a turning point. [4]

For Examiner's Use

13 The cost in thousands of dollars, C, of producing n hundred pairs of a certain type of running shoes is given by the formula $C = 1.2n^2 - 14.4n + 53.7$.

(a) Find the initial cost of production. [1]

(b) Express C in the form $a(x + h)^2 + k$ where a, h and k are constants to be determined. [3]

(c) Explain the meaning of the values of h and k in the formula as expressed in **part (b)**. [1]

(d) Find the maximum number of pairs of running shoes for which the cost will be [3]

[Turn over

For Examiner's Use

50 thousand dollars.

For Examiner's Use

14 The line $y = -x + 4$ is a tangent to a circle at the point A and the centre of the circle is at $B(-4, -2)$.

(a) Find coordinates of A. [3]

(b) Find the radius and the equation of the circle. [3]

[Turn over

For Examiner's Use

(c) Find the equation of another tangent to the circle which is parallel to $y = -x + 4$. [4]

For Examiner's Use

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For Examiner's Use

Bedok View Secondary School
Department of Mathematics — Mark Scheme

Year	2025	**Level & Stream**	4E
Type of Assessment	Preliminary Exam	**Subject & Paper**	Additional Math P2

Qns | **Working**

1a

$$\mathrm{N}(x) = x^2 \ln\left(\frac{2}{x}\right)$$

let $u = x^2$ and $v = \ln\left(\frac{2}{x}\right)$

$$\frac{\mathrm{d}u}{\mathrm{d}x} = 2x \qquad v = \ln 2 - \ln x$$

$$\frac{\mathrm{d}v}{\mathrm{d}x} = -\frac{1}{x}$$

$$\mathrm{N}'(x) = x^2.\left(-\frac{1}{x}\right) + \ln\left(\frac{2}{x}\right).2x$$

$$= -x + 2x\ln\left(\frac{2}{x}\right)$$

b

stationary point $\mathrm{N}'(x) = 0$

$$-x + 2x\ln\left(\frac{2}{x}\right) = 0$$

$$x\left(-1 + 2\ln\left(\frac{2}{x}\right)\right) = 0$$

$x = 0$ (rej) or $-1 + 2\ln\left(\frac{2}{x}\right) = 0$

$$2\ln\left(\frac{2}{x}\right) = 1$$

$$\ln\left(\frac{2}{x}\right) = \frac{1}{2}$$

$$e^{0.5} = \frac{2}{x}$$

$$x = 2 \div e^{0.5}$$

$$x = 1.21306$$

x	1.1	1.21306	1.3
Sign of N'(x)	+ve	0	−ve
Sketch of tangent			

$\therefore$ max absorption rate at x = 1.21 (3 s.f.)

Qns	Working
2a	$T = 22 + Ae^{kt}$ when $t = 0,\ T = 90$ $90 = 22 + Ae^{k(0)}$ $90 = 22 + A$ $A = 68$
b	when $t = 3.5,\ T = 79$ $79 = 22 + 68e^{k(3.5)}$ $57 = 68e^{k(3.5)}$ $\frac{57}{68} = e^{k(3.5)}$ $\ln\left(\frac{57}{68}\right) = k(3.5)$ $k = \ln\left(\frac{57}{68}\right) \div 3.5$ $k = -0.050416$ $k = -0.0504$ (3 s.f.)
c	$T = 22 + 68e^{-0.050416t}$ when $t = 10$, $T = 22 + 68e^{-0.050416(10)}$ $= 63.0728$ Since 63.0728 °C is between 55 °C and 70 °C, the coffee is suitable to be served.
d	After a long time, T approaches 22 °C.

Qns	Working
3a	$\cos\theta = \dfrac{ED}{14}$ $ED = 14\cos\theta$ Let G be the point on FD such that $BG \perp FD$ $\sin\theta = \dfrac{EG}{3}$ $EG = 3\sin\theta$ $FD = 14\cos\theta + 3\sin\theta + 6$ (shown)
b	$R = \sqrt{14^2 + 3^2}$ $= \sqrt{205}$ $\tan\alpha = \dfrac{3}{14}$ $\alpha = 12.0947$ $FD = \sqrt{205}\cos(\theta - 12.1°) + 6$ (3 s.f.)
c	$\sqrt{205}\cos(\theta - 12.0947°) + 6 = 12$ $\cos(\theta - 12.0947°) = \dfrac{6}{\sqrt{205}}$ $\theta - 12.0947° = 65.2248°$ $\theta = 77.3195°$ $\theta = 77.3°$ (1 d.p.)
d	Maximum $\sqrt{205}\cos(\theta - 12.0947°) + 6$ $= \sqrt{205} + 6$ $= 20.317$ $\therefore$ it is possible for FD to be 15 cm Or $\sqrt{205}\cos(\theta - 12.0947°) + 6 = 15$ $\cos(\theta - 12.0947°) = \dfrac{9}{\sqrt{205}}$ $\theta - 12.0947° = 51.054°$ $\theta = 63.1°$ (1 d.p.) FD is 15 cm when $\theta = 63.1°$

Qns	Working
4a	

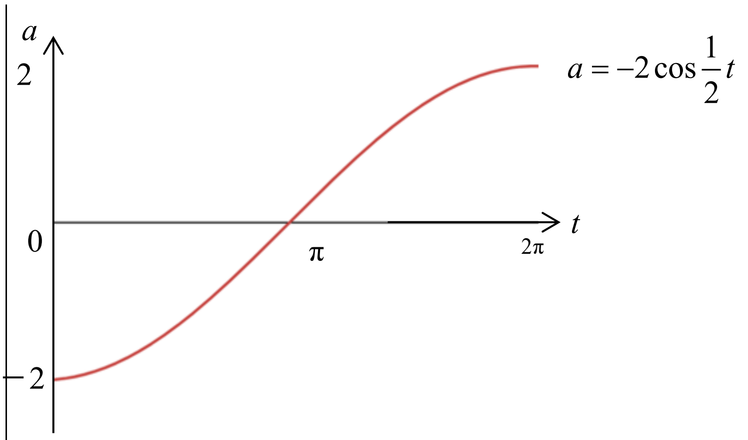
Qns	Working
b	$N = a10^{kt}$ $\lg N = \lg a10^{kt}$ $\lg N = \lg a + \lg 10^{kt}$ $\lg N = \lg a + kt$ $\lg N = kt + \lg a$ $k = \text{gradient}$ $= 0.232$ (3 s.f.) ± 0.1 $\lg a = \text{vertical axis} - \text{intercept}$ $\lg a = 2.5 \pm 0.05$ $a = 10^{2.5}$ $=316$ (3 s.f.)
5a	$-2(x+1)(x-2)(x-1.5)$ $= (x^2 - x - 2)(-2x + 3)$ $= -2x^3 + 3x^2 + 2x^2 - 3x + 4x - 6$ $= -2x^3 + 5x^2 + x - 6$
b	Quotient: $x - 3$ $x^2 + 0x - 1 \overline{) x^3 - 3x^2 - 2x + 5}$ $-(x^3 + 0x^2 - x)$ $-3x^2 - x + 5$ $-(-3x^2 + 0x + 3)$ $-x + 2$ $\therefore$ Quotient $= x - 3$

Qns	Working
c	$hx^3 - 7x^2 + kx - 2 = \left(3x^2 - 5x - 2\right) \times \text{Quotient}$ $hx^3 - 7x^2 + kx - 2 = (3x+1)(x-2) \times \text{Quotient}$ sub $x = -\frac{1}{3}$, $h\left(-\frac{1}{3}\right)^3 - 7\left(-\frac{1}{3}\right)^2 + k\left(-\frac{1}{3}\right) - 2 = 0$ $-\frac{1}{27}h - \frac{1}{3}k = \frac{25}{9}$----------(1) sub $x = 2$, $h(2)^3 - 7(2)^2 + k(2) - 2 = 0$ $8h + 2k = 30$----------(2) From (2), $4h + k = 15$ $k = 15 - 4h$----------(3) sub (3) into (1), $-\frac{1}{27}h - \frac{1}{3}(15 - 4h) = \frac{25}{9}$ $-\frac{1}{27}h - 5 + \frac{4}{3}h = \frac{25}{9}$ $\frac{35}{27}h = \frac{70}{9}$ $h = 6$ sub $h = 6$ into (3), $k = 15 - 4(6)$ $k = -9$

Qns	Working
6a	$AB = AE$ (tangent from external point) $AC = AD$ (tangent from external point) $\frac{AB}{AC} = \frac{AE}{AD}$ (ratio of corresponding sides are equal) $\angle BAE = \angle CAD$ (common angle) ΔABE is similar to ΔACD (ratio of 2 pairs of corresponding sides and a pair of included $\angle$s are equal)
b	$\angle ABE = \angle ACD$ (ΔABE is similar to ΔACD) $\angle ACD = \angle ADC$ (base $\angle$ of isosceles Δ) $\angle EBC = 180° - \angle ABE$ (adjacent $\angle$s on a straight line) Since $\angle ADC + \angle EBC = 180°$, by the converse of angles in opposite segments, a circle can be drawn to pass through $BEDC$.
c	Let $\angle CBF = x$, $\angle BEF = x$ (alternate segment thm) $BE // CD$ since ΔABE is similar to ΔACD $\angle ECD = x$ (alternate angles, $BE // CD$) $\angle EDF = x$ (alternate segment thm) $\therefore \angle CBF = \angle EDF$ (proven)

Qns	Working
7a	General term of $\left(1-\frac{x}{5}\right)^8$ $=\binom{8}{r}(1)^{8-r}\left(-\frac{x}{5}\right)^r$ $=\binom{8}{r}\left(-\frac{x}{5}\right)^r$ $(7+2x)\left(1-\frac{x}{5}\right)^8$ $=(7+2x)\left(...+\binom{8}{3}\left(-\frac{x}{5}\right)^3+\binom{8}{2}\left(-\frac{x}{5}\right)^2+...\right)$ x^3 term $=7\binom{8}{3}\left(-\frac{x}{5}\right)^3+2x\binom{8}{2}\left(-\frac{x}{5}\right)^2$ $=7(56)\left(-\frac{x^3}{125}\right)+2x(28)\left(\frac{x^2}{25}\right)$ $=-\frac{392}{125}x^3+\frac{56}{25}x^3$ Coefficient of $x^3=-\frac{112}{125}$
b	$\binom{n}{2}3^{n-2}\left(\frac{1}{2}x\right)^2=\binom{n}{2}3^{n-2}\frac{1}{4}x^2$ $\binom{n}{3}3^{n-3}\left(\frac{1}{2}x\right)^3=\binom{n}{3}3^{n-3}\frac{1}{8}x^3$ $\binom{n}{2}3^{n-2}\frac{1}{4}=3\times\binom{n}{3}3^{n-3}\frac{1}{8}$ $\binom{n}{2}3^{n-2}.2=\binom{n}{3}3^{n-2}$ $\binom{n}{2}.2=\binom{n}{3}$ $\frac{n(n-1)}{2}.2=\frac{n(n-1)(n-2)}{2\times 3}$ $6=n-2$ $n=8$

Qns	Working
8a	$\cos 2A = \frac{1}{3}$ $1 - 2\sin^2 A = \frac{1}{3}$ $\sin^2 A = \frac{1}{3}$ $\sin A = \frac{1}{\sqrt{3}}$ or $-\frac{1}{\sqrt{3}}$ (rej) $\text{cosec } A = \frac{1}{\sin A}$ $= 1 \div \frac{1}{\sqrt{3}}$ $= \sqrt{3}$
b	$\sin 105°$ $= \sin(45° + 60°)$ $= \sin 45° \cos 60° + \cos 45° \sin 60°$ $= \frac{1}{\sqrt{2}} . \frac{1}{2} + \frac{1}{\sqrt{2}} . \frac{\sqrt{3}}{2}$ $= \frac{1+\sqrt{3}}{\sqrt{2}.2}$ $\text{c osec } A - \sqrt{2} \sin 105°$ $= \sqrt{3} - \sqrt{2} . \frac{1+\sqrt{3}}{\sqrt{2}.2}$ $= \sqrt{3} - \frac{1+\sqrt{3}}{2}$ $= \frac{\sqrt{3}-1}{2}$

Qns	Working
9a	
b	$v=\int -2\cos\frac{1}{2}t\ \mathrm{d}t$ $=\dfrac{-2\sin\frac{1}{2}t}{\frac{1}{2}}+c$ $=-4\sin\frac{1}{2}t+c$ Given $t=0$ and $v=3$, find c $3=-4\sin\frac{1}{2}(0)+c$ $c=3$ $v=-4\sin\frac{1}{2}t+3$ find t when $v=0$, $-4\sin\frac{1}{2}t+3=0$ $\sin\frac{1}{2}t=\frac{3}{4}$ basic $\angle$ of $\frac{1}{2}t=0.84806$ $t=1.6961$ $t=1.70$ (3 s.f.)

Qns	Working
c	basic $\angle$ of $\frac{1}{2}t = 0.84806$ since $\sin\frac{1}{2}t$ is +ve, $\frac{1}{2}t$ is also in 2nd quad. $\frac{1}{2}t = \pi - 0.84806$ $\frac{1}{2}t = 2.2935$ $t = 4.5870$ Displacement $= \int \left(-4\sin\frac{1}{2}t + 3 \right) \mathrm{d}t$ $= -4.\frac{-\cos\frac{1}{2}t}{\frac{1}{2}} + 3t + C_1$ $= 8\cos\frac{1}{2}t + 3t + C_1$ Distance travelled in first 5 min $= \left\| \left[8\cos\frac{1}{2}t + 3t \right]_0^{1.6961} \right\| + \left\| \left[8\cos\frac{1}{2}t + 3t \right]_{1.6961}^{4.5870} \right\| + \left\| \left[8\cos\frac{1}{2}t + 3t \right]_{4.5870}^{5} \right\|$ $= \|10.379 - 8\| + \|8.4696 - 10.379\| + \|8.5908 - 8.4696\|$ $= 2.3798 + 1.9100 + 0.1212$ $= 4.41$ m

Qns	Working
10a	$y = 2x - 1 - - - - - (1)$ $y = -x^{\frac{3}{2}} - - - - - - - - (2)$ fr (2), $y^2 = x^3 - - - (3)$ sub (1) into (3), $(2x-1)^2 = x^3$ $4x^2 - 4x + 1 = x^3$ $0 = x^3 - 4x^2 + 4x - 1$ $(x-1)$ is a factor since $A(1,1)$ is intersection $0 = (x-1)(Ax^2 + Bx + C)$ $x^3 - 4x^2 + 4x - 1 = (x-1)(Ax^2 + Bx + C)$ compare x^3 : $1 = A$ compare constant: $-1 = -C$ $1 = C$ sub $x = 2$, $-1 = 2^2 + 2B + 1$ $-3 = B$ $(x-1)(x^2 - 3x + 1) = 0$ $x - 1 = 0$ or $x^2 - 3x + 1 = 0$ $x = 1$ $x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(1)}}{2(1)}$ $= 2.6180$ or 0.381966 sub $x = 0.381966$, $y = 2x - 1$ $y = 2(0.381966) - 1$ $= -0.236068$ $\therefore C = (0.38197, -0.23607)$ (shown)

Qns	Working
b	sub $y=0$ to find $x-$coordinate of B $y=2x-1$ $0=2x-1$ $x=0.5$ Let A be the shaded area above the x-axis Let B be the shaded area below the x-axis $A=\left\|\int_0^1 x^{\frac{3}{2}}\mathrm{d}x\right\|-\frac{1}{2}(0.5)(1)$ $=\left[\frac{x^{\frac{5}{2}}}{\frac{5}{2}}\right]_0^1-0.25$ $=\frac{2}{5}-0.25$ $= 0.15$ units^2 $B=\int_0^{0.38197} -x^{\frac{3}{2}}\mathrm{d}x+\frac{1}{2}(0.5-0.38197)(0.23607)$ $=\left\|\left[-\frac{x^{\frac{5}{2}}}{\frac{5}{2}}\right]_0^{0.38197}\right\|+0.013931$ $= 0.03606089 + 0.0139391$ $= 0.049999$ units^2 Total shaded area $= 0.15+0.049999$ $= 0.200$ units^2 (3 s.f.)

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \cdots + \binom{n}{r} a^{n-r} b^r + \cdots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

[Turn over

1 The nutrient absorption rate, N, in mg/hour, of a plant depends on the diameter x, in mm, of its root hairs and is modelled by the function $\mathrm{N}(x) = x^2 \ln\left(\frac{2}{x}\right)$, where $0 < x < 2$.

(a) Find $\mathrm{N}'(x)$. [3]

(b) Find the optimal root hair diameter that maximises nutrient absorption and show that the nutrient absorption is a maximum. [5]

For Examiner's Use

2 Hot coffee is heated to a temperature of $90\ °\mathrm{C}$. The temperature, $T\ °\mathrm{C}$, t minutes after removal from the heat source is given by $T = 22 + Ae^{kt}$, where A and k are constants.

(a) Explain why $A = 68$. [1]

(b) After 3.5 minutes, the temperature of coffee is $79\ °\mathrm{C}$. Find k. [3]

[Turn over

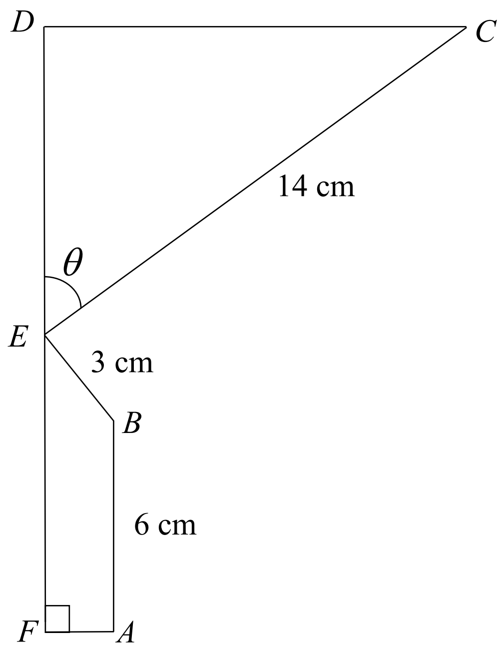
For Examiner's Use

(c) The coffee should only be served when it is between $55\ °\text{C}$ to $70\ °\text{C}$. Determine, with working, whether the coffee is suitable to be served 10 minutes after removal from the heat source. [2]

(d) After a long time, T approaches a value. Calculate this value. [1]

For Examiner's Use

3 In the diagram, angle $CED = \theta$. EC is perpendicular to BE. The lengths of AB, BE and EC are 6 cm, 3 cm and 14 cm respectively. AB is parallel to FED and CD is parallel to AF.



(a) Show that $FD = 14\cos\theta + 3\sin\theta + 6$. [3]

[Turn over

For Examiner's Use

(b) Express FD in the form $R\cos(\theta-\alpha)+d$ where $R>0$, $0°<\alpha<90°$, and d is a constant. [3]

(c) Find the value of θ when $FD=12\,\text{cm}$. [2]

(d) Determine if it is possible for FD to be 15 cm. [2]

For Examiner's Use

4 In a biology experiment, a certain type of bacteria multiplies rapidly. The number of bacteria, N, after t hours is modelled by the formula $N = a10^{kt}$, where a and k are constants. The table below shows corresponding values of t and N.

t	2	4	6	8
N	900	2700	8100	24 300

(a) On the grid on the next page, plot $\lg N$ against t and draw a straight line graph to illustrate the information. [4]

[Turn over

For Examiner's Use

(b) Use the graph to estimate the value of each of the constants a and k. [2]

For Examiner's Use

5 **(a)** The cubic polynomial $f(x)$ is such that the coefficient of x^3 is -2 and the roots of $f(x)=0$ are -1, 2 and $\frac{3}{2}$. Find the expression of $f(x)$ in descending powers of x. [3]

(b) Find the quotient when x^3-3x^2-2x+5 is divided by x^2-1. [2]

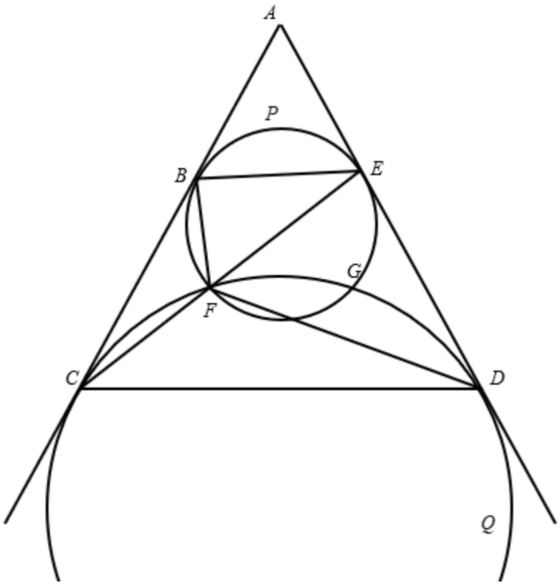
(c) The expression hx^3-7x^2+kx-2, where h and k are constants, has a factor $3x^2-5x-2$. Find the value of h and k. [5]

[Turn over

For Examiner's Use

For Examiner's Use

6 The diagram shows two circles P and Q which intersect each other at F and G. The line ABC is tangent to P and Q at B and C respectively. The line AED is tangent to P and Q at E and D respectively. EFC is a straight line.



(a) Prove that triangle ABE is similar to triangle ACD. [4]

For Examiner's Use

(b) Explain why a circle can be drawn to pass through *BEDC*. [3]

(c) Prove that angle *CBF* = angle *EDF*. [3]

For Examiner's Use

7 **(a)** Find the coefficient of x^3 in the expansion of $(7+2x)\left(1-\frac{x}{5}\right)^8$. [4]

For Examiner's Use

(b) In the expansion of $\left(3+\frac{x}{2}\right)^n$, the coefficient of x^2 is three times the coefficient of x^3.

Find n. [4]

For Examiner's Use

8 **Do not use a calculator in this question.**

(a) Find the exact value of cosec A when $\cos 2A = \dfrac{1}{3}$ and A is acute. [4]

[Turn over

For Examiner's Use

(b) Hence, find the exact value of $\operatorname{cosec} A - \sqrt{2}\sin 105°$. [5]

For Examiner's Use

9 A particle moves in a straight line so that its acceleration, a m/s^2, is given by $a = -2\cos\frac{1}{2}t$, where t is the time in seconds after leaving a fixed point O.

(a) Sketch the acceleration-time graph of the particle for $0 \le t \le 2\pi$. [2]

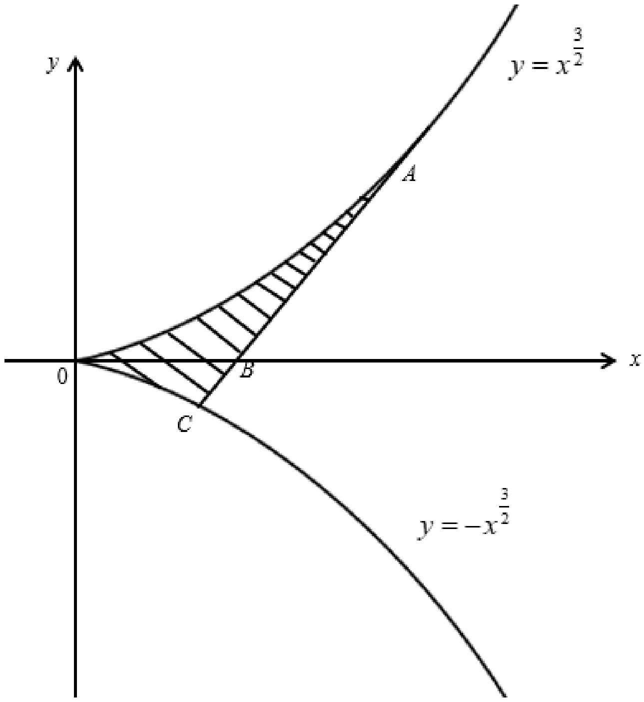
(b) The initial velocity of the particle is 3 m/s. Find the earliest time when the particle first comes to rest. [4]

Continuation of working space for question **9(b)**.

(c) Find the total distance travelled in the first 5 seconds. [5]

For Examiner's Use

10 The diagram shows the graphs of $y = x^{\frac{3}{2}}$ and $y = -x^{\frac{3}{2}}$. The line $y = 2x - 1$ intersects the graph of $y = x^{\frac{3}{2}}$ at $A(1, 1)$, the x-axis at B, and the graph of $y = -x^{\frac{3}{2}}$ at C.



(a) Show that the coordinates of C are $(0.38197, -0.23607)$, correct to 5 significant figures. [5]

For Examiner's Use

Continuation of working space for question **10(a)**.

(b) Find the area of the shaded region. [6]

For Examiner's Use

Continuation of working space for question **10(b)**.

End of Paper

[Turn over

For Examiner's Use

BEDOK SOUTH SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2025

4E5N

CANDIDATE NAME | CLASS | REGISTER NUMBER

ADDITIONAL MATHEMATICS
Paper 1

4049 / 01
2 hours 15 minutes

90

Answer **all** the questions.

1 Prove that $\sin 2x - \cos 2x \tan x = \tan x$. [4]

2 Show that the solution of $2^{3x+4} \times 5^{2x+1} = 5^{3(x+1)} \times 16^x$ is $2\lg\frac{4}{5}$. [4]

3 Giving your answer in the form $\frac{c+d\sqrt{2}}{3}$, solve, without using a calculator,
$$x\sqrt{18} = 3x + \sqrt{32}.$$ [5]

4 A square of area $(11+\sqrt{120})\text{ cm}^2$ has a length of $(\sqrt{a}+\sqrt{b})$ cm, where $a < b$.
Without using a calculator, find the values of *a* and *b*. [5]

5 The line $3x + 4y = 13$ intersects the curve $y = \frac{6x-10}{3x-1}$ at points *P* and *Q*.
Calculate the exact length of the line segment *PQ*. [5]

6 The coefficient of x^3 in the expansion of $(10+kx)\left(2-\frac{1}{2}x\right)^8$ is zero.
Find the value of the constant *k*. [5]

7 Solve the equation $\frac{5\cos x + \sin 2x}{\cos 2x + 5\sin x} = \cot x$ for $0 \le x \le 180°$. [6]

8 **(a)** Show that the derivative of $x(x-3)^4$ with respect to *x* is $(5x-3)(x-3)^3$. [2]

(b) Hence, given that $y = \frac{x(x-3)^4}{5x-3}$ and y is decreasing at a constant rate of 70 units/s, calculate the rate of change of *x* when $x = 1$. [4]

9 **(a)** Express $y = 5 - 12x - 3x^2$ in the form $y = a(x+b)^2 + c$ and hence show that y can never be greater than 20. [3]

(b) Explain why there are no values of k for which the curve $y = (k-1)x^2 + 2(k+2)x + k + 3$ is always positive. [4]

10 A rectangular field has sides $(3x-5)$ m and $(x-10)$ m. Its area is at most 200 m^2.

(a) Find the range of values of x that satisfies the above sides and area conditions. [4]

(b) Justify whether a fence of 98 m is enough to enclose the field. [3]

11 **(a)** State the range of values of x for which the equation below is valid.

$$2\log_3(4-x) - \log_9(x-4)^2 = 2$$ [1]

(b) Express $2\log_3(4-x) - \log_9(x-4)^2 = 2$ as a quadratic equation $x^2 + bx + c = 0$ and explain why there is only one real solution. [6]

12 A quadratic curve is given by $y = hx^2 - 2x - 9h + 6$, where h is a constant.

(a) Show that the equation $y = 0$ has real roots for all values of h except $h = 0$. [3]

(b) State the value of h in the case where $y = 0$ has two real and equal roots. [1]

(c) Given that the line $y = 2x - h - 12$ meets the curve $y = hx^2 - 2x - 9h + 6$, find the range of values of h. [5]

13 It is given that $f(x) = x^2 e^{x+2}$.

(a) Show that the range of values of x for which $f(x)$ is a decreasing function is $-2 < x < 0$. [4]

(b) The gradient with the least value is in the range $-2 < x < 0$.
Find the value of this gradient, giving your answer in exact form. [5]

14 It is given that $f(x) = 3x^3 + 2x^2 + 16$. The remainder when $f(x)$ is divided by $x - 3a$, where a is a constant, is the same as the remainder when it is divided by $3x + 2$.

(a) Find the possible values of a. [3]

(b) Show, with clear working, that $x = -2$ is a solution of $f(x) = 0$. [1]

(c) Explain why $f(x) = 0$ has only one real root. [5]

(d) Hence use your answers to parts (b) and (c) to solve the equation

$$16y^3 + 2y - 3 = 0.$$ [2]

END OF PAPER

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!\,r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\cos ec^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for $\Delta\, ABC$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

ANSWERS

(Bedok South) AM4 P1 (2025) Students

1. **Hint:** Convert to $\sin x$ and $\cos x$ by apply double angle formulae.

2. $2\lg\left(\frac{4}{5}\right)$

3. $\frac{8+4\sqrt{2}}{3}$

4. $a = 5,\ \ b = 6$

5. $PQ = 5$ units

6. $k = 5$

7. $30°,\ \ 90°,\ \ 150°$

8. (a) **Hint:** Apply Product Rule $\frac{\mathrm{d}(uv)}{\mathrm{d}x} = \frac{\mathrm{d}u}{\mathrm{d}x}v + u\frac{\mathrm{d}v}{\mathrm{d}x}$

(b) 2.5 units/s

9. (a) $y = -3(x+2)^2 + 17$

(b) **Hint:** Apply the two conditions:
(i) coefficient of $x^2 > 0$
(ii) discriminant < 0

10.(a) $10 < x \leq 15$

(b) It is enough to enclose the field.

11.(a) $x < 4$

(b) $x = -5$

12.(a) **Hint:** Apply completing the square to the discriminant.

(b) $h = \frac{1}{3}$

(c) $h \leq \frac{1}{4}$ or $h \geq 2$ and $h \neq 0$

13.(a) **Hint:** For decreasing function, $f'(x) < 0$

(b) $\left(2 - 2\sqrt{2}\right)e^{\sqrt{2}}$

14.(a) $a = 0$ or $a = -\frac{2}{9}$

(b) **Hint:** Apply Factor Theorem

(c) **Hint:** Show that discriminant < 0

(d) $y = \frac{1}{2}$

MARKING SCHEME

(Bedok South) 4E5N AM P1 (Prelim 2025) (w Ans)

Qn	Solution	Marks	Testing
1	**Trigonometric Identities**		
	LHS		
	$= \sin 2x - \cos 2x \tan x$		
	$= 2\sin x\cos x - (2\cos^2 x - 1)\dfrac{\sin x}{\cos x}$	M2	Apply double angle for sine & cosine
	$= 2\sin x\cos x - 2\sin x\cos x + \dfrac{\sin x}{\cos x}$	M1	Expand & simplify
	$= \tan x$	A1	
	$= RHS$		
2	**Exponential Equations**		
	$2^{2(x+2)} \times 5 = 5^{x+3} \times 8^x$		
	$2^4(2^{2x}) \times 5 = 5^3(5^x) \times 2^{3x}$	M1	Split the powers
	$\dfrac{2^{2x}}{(2^{3x})(5^x)} = \dfrac{5^3}{2^4 \times 5}$	M1	Separate factors with x and non-x
	$\dfrac{1}{(2^x)\,(5^x)} = \dfrac{25}{16}$		
	$\dfrac{1}{10^x} = \left(\dfrac{5}{4}\right)^2$		Combine common index
	$10^x = \left(\dfrac{4}{5}\right)^2$		
	$x = \lg\left(\dfrac{4}{5}\right)^2$	M1	Express in logarithm
	$\boldsymbol{x = 2\lg\left(\dfrac{4}{5}\right)}$	A1	Power law
	OR		
	$2^{2(x+2)} \times 5 = 5^{x+3} \times 8^x$		
	$2^{2x+4} \times 5 = 5^{x+3} \times 2^{3x}$	M1	Convert to prime bases
	$\dfrac{2^{2x+4}}{2^{3x}} = \dfrac{5^{x+3}}{5}$	M1	Separate factors of different bases
	$\dfrac{2^4}{2^x} = 5^x 5^2$		
	$10^x = \dfrac{2^4}{5^2}$		
	$10^x = \dfrac{4^2}{5^2}$		Combine common index
	$x = \lg\left(\dfrac{4}{5}\right)^2$	M1	Express in logarithm
	$\boldsymbol{x = 2\lg\left(\dfrac{4}{5}\right)}$	A1	Power law
3	**Surds Expressions**		
	$x\sqrt{18} = 3x + \sqrt{32}$		
	$3x\sqrt{2} - 3x = 4\sqrt{2}$		
	$3x(\sqrt{2} - 1) = 4\sqrt{2}$	M1	Grouping
	$3x = \dfrac{4\sqrt{2}}{\sqrt{2} - 1}$		
	$= \dfrac{4\sqrt{2}\ (\sqrt{2} + 1)}{(\sqrt{2} - 1)(\sqrt{2} + 1)}$	M1	Rationalise denominator with conjugate
	$= \dfrac{8 + 4\sqrt{2}}{(\sqrt{2})^2 - (1)^2}$	M1	Expansion
		M1	Difference of squares
	$x = \boldsymbol{\dfrac{8 + 4\sqrt{2}}{3}}$	A1	
	OR		
	$x\sqrt{18} = 3x + \sqrt{32}$		
	$3\sqrt{2}x = 3x + 4\sqrt{2}$		
	$(3\sqrt{2} - 3)x = 4\sqrt{2}$	M1	Grouping
	$x = \dfrac{4\sqrt{2}}{3\sqrt{2} - 3}$		
	$= \dfrac{4\sqrt{2}\ (3\sqrt{2} + 3)}{(3\sqrt{2} - 3)(3\sqrt{2} + 3)}$	M1	Rationalise denominator with conjugate
	$= \dfrac{24 + 12\sqrt{2}}{(3\sqrt{2})^2 - (3)^2}$	M1	Expansion
		M1	Difference of squares
	$= \dfrac{24 + 12\sqrt{2}}{9}$		
	$= \boldsymbol{\dfrac{8 + 4\sqrt{2}}{3}}$	A1	
4	**Surds (Application)**		
(a)	Area of square $= 11 + \sqrt{120}$		
	$(\sqrt{a} + \sqrt{b})^2 = 11 + 2\sqrt{30}$		
	$a + b + 2\sqrt{ab} = 11 + 2\sqrt{30}$	M1	Expand
	Equating the rational part,	M1	Equating rational & irrational parts separately
	$a + b = 11$		
	$b = 11 - a\ldots\ldots(1)$		
	Equating the irrational part,		
	$2\sqrt{ab} = 2\sqrt{30}$		
	$ab = 30\ \ldots\ldots(2)$		
	Subst. (1) into (2),		
	$a(11 - a) = 30$	M1	Substitution method
	$11a - a^2 = 30$		
	$a^2 - 11a + 30 = 0$		
	$(a - 5)(a - 6) = 0$		
	$a = 5$ or $a = 6$	M1	Solve quadratic equation
	Corresponding b values:		
	$b = 6$ or $b = 5$		
	Since $a < b$		
	$\boldsymbol{a = 5,\ \ b = 6}$	A1	Correct set of values

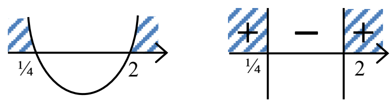
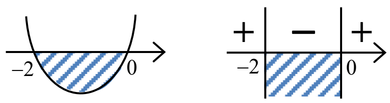
Qn	Solution	Marks	Testing
5	**Simultaneous Equations**		
	$4x - y = 5 \quad \dots\dots [1]$		
	$y = 2x^2 - 6x + 7 \quad \dots\dots [2]$		
	Subst (2) into (1)		
	$4x - (2x^2 - 6x + 7) = 5$	M1	Substitution or Elimination Method
	$4x - 2x^2 + 6x - 7 = 5$		
	$2x^2 - 10x + 12 = 0$		
	$x^2 - 5x + 6 = 0$		
	$(x-2)(x-3) = 0$	M1	Factorisation or Quadratic Formula
	$x = 2 \quad or \quad x = 3$		
	Subst respective x values into [1],		
	$y = 4(2) - 5$, $y = 3$ | $y = 4(3) - 5$, $y = 7$	M1	Find values of 2nd variable
	$P(2, 3)$ and $Q(3, 7)$		
	$PQ = \sqrt{(3-2)^2 + (7-3)^2}$	M1	Apply Distance formula
	$PQ = \sqrt{17}$	A1	
6	**Binomial Theorem**		
	General term of expansion $\left(2 - \frac{1}{2}x\right)^8$ is		
	$T_{r+1} = \binom{8}{r}(2)^{8-r}\left(-\frac{x}{2}\right)^r$		
	$= \binom{8}{r}(-1)^r(2)^{8-2r}\,x^r$	M1	General term of binomial expansion
	$T_3 = \binom{8}{2}(-1)^2(2)^{8-2(2)}\,x^2$		
	$= 448x^2$	A1	Extract specific terms
	$T_4 = \binom{8}{3}(-1)^3(2)^{8-2(3)}\,x^3$		
	$= -224x^3$	A1	
	$(10 + kx)\left(2 - \frac{1}{2}x\right)^8$		
	$= (10 + kx)(\dots 448x^2 - 224x^3 \dots)$		
	Coeff. of $x^3 = 0$		
	$(10)(-224) + k(448) = 0$	M1	Equating coefficient
	$448k = 2240$		
	$k = 5$	A1	
7	**Trigonometric Equations**		
	$\frac{5\cos x + \sin 2x}{\cos 2x + 5\sin x} = \cot x$		
	$\frac{5\cos x + \sin 2x}{\cos 2x + 5\sin x} = \frac{\cos x}{\sin x}$	M1	Convert to $\sin x$ & $\cos x$
	$\sin x\,(5\cos x + \sin 2x)$		
	$= \cos x\,(\cos 2x + 5\sin x)$		

Qn	Solution	Marks	Testing
	$\sin x\,(5\cos x + 2\sin x\cos x)$	M1	Apply double angle formula $\sin 2x$ & $\cos 2x$
	$= \cos x\,(\cos^2 x - \sin^2 x + 5\sin x)$		
	$5\sin x\cos x + 2\sin^2 x\cos x$		
	$= \cos^3 x - \sin^2 x\cos x + 5\sin x\cos x$		
	$2\sin^2 x\cos x = \cos^3 x - \sin^2 x\cos x$		
	$3\sin^2 x\cos x - \cos^3 x = 0$	M1	Expand & simplify
	$\cos x\,(3\sin^2 x - \cos^2 x) = 0$		
	$\cos x = 0$ | $3\sin^2 x - \cos^2 x = 0$	M1	Zero product rule
	$x = 90°$ | $3\sin^2 x = \cos^2 x$		
	$3\tan^2 x = 1$		
	$\tan^2 x = \frac{1}{3}$		
	$\tan x = \pm\frac{1}{\sqrt{3}}$		Either solution correct
	$x = 30°,\ 150°$	A1	
	$\therefore x = 30°,\ 90°,\ 150°$	A1	Final solution
8	**Differentiation (Product Rule)**		
(a)	$\frac{d}{dx}[x(x-3)^4]$		
	$= (x-3)^4 + 4x(x-3)^3$	M1	Apply Product Rule
	$= (x-3)^3(x - 3 + 4x)$		
	$= (5x-3)(x-3)^3$	A1	
	Differentiation (Quotient Rule)		
(b)	$y = \frac{x(x-3)^4}{5x-3}$		
	$\frac{dy}{dx} = \frac{(5x-3)(x-3)^3(5x-3) - 5x(x-3)^4}{(5x-3)^2}$	M1	Apply Quotient Rule
	$= \frac{(x-3)^3\left((5x-3)^2 - 5x(x-3)\right)}{(5x-3)^2}$		
	$= \frac{(x-3)^3(25x^2 - 30x + 9 - 5x^2 + 15x)}{(5x-3)^2}$		
	$= \frac{(x-3)^3(20x^2 - 15x + 9)}{(5x-3)^2}$	M1	Factorise & simplify
	When $x = 1$,		
	$\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$		
	$-70 = \frac{(x-3)^3(20x^2 - 15x + 9)}{(5x-3)^2} \times \frac{dx}{dt}$		
	$= \frac{(1-3)^3(20 - 15 + 9)}{(5-3)^2} \times \frac{dx}{dt}$	M1	Rate of change
	$-70 = -28 \times \frac{dx}{dt}$		
	$= 2.5$ units/s	A1	

Qn	Solution	Marks	Testing
9	**Quadratic Functions**		
(a)	$y = 5 - 12x - 3x^2$		
	$= -3x^2 - 12x + 5$		
	$= -3(x^2 + 4x) + 5$		
	$= -3\{(x+2)^2 - (2)^2\} + 5$	**M1**	Completing the Square
	$= -3(x+2)^2 + 12 + 5$		
	$\boldsymbol{y = -3(x+2)^2 + 17}$	**A1**	
	Since $(x+2)^2 \geq 0$ $-3(x+2)^2 \leq 0$ $-3(x+2)^2 + 17 \leq 17$ max $y = 17 < 20$, ∴ for all values of y, it can never be greater than 20.	**B1**	
(b)	Given $y = (k-1)x^2 + 2(k+2)x + k + 3$		
	For y to be always positive, (i) Coeff of $x^2 > 0$, and (ii) Discriminant < 0.		2 conditions for $y > 0$
	From (i), $k - 1 > 0$ $k > 1$	**D1**	
	From (ii), $\{2(k+2)\}^2 - 4(k-1)(k+3) < 0$	**M1**	
	$4(k+2)^2 - 4(k-1)(k+3) < 0$ $(k+2)^2 - (k-1)(k+3) < 0$ $(k^2 + 4k + 4) - (k^2 + 2k - 3) < 0$ $2k + 7 < 0$		$D = b^2 - 4ac$
	$k < -\frac{7}{2}$	**D1**	
	From the two inequalities, $k > 1$ and $k < -\frac{7}{2}$, there are no values of k for which $y > 0$.	**A1**	
10	**Quadratic Inequalities (Word Problem)**		
(a)	Since $Side > 0$, $3x - 5 > 0$ and $x - 10 > 0$ $x > \frac{5}{3}$ and $x > 10$ ......(1)	**M1**	Form Side inequalities
	$Area\ of\ rectangle \leq 200$ $(3x-5)(x-10) \leq 200$	**M1**	Form Area inequality
	$3x^2 - 35x + 50 \leq 200$ $3x^2 - 35x - 150 \leq 0$		
	$(3x+10)(x-15) \leq 0$ $-\frac{10}{3} \leq x \leq 15$ ......(2)	**M1**	Factorisation
	From (1) & (2), $\boldsymbol{10 < x \leq 15}$	**A1**	

Qn	Solution	Marks	Testing
(b)	To enclose the field, $Perimeter \leq 98$		
	$2(3x-5) + 2(x-10) \leq 98$	**M1**	Form Perimeter inequality
	$2(4x - 15) \leq 98$ $4x - 15 \leq 49$		
	$x \leq 16$ ......(3)	**A1**	
	From part (a) $10 < x \leq 15$ and (3) $x \leq 16$, it is **enough** to enclose the field.	**B1**	
11	**Logarihemic Equations**		
(a)	For a valid logarithm, $4 - x > 0$ $\therefore\ x < 4$	**B1**	
(b)	$2\log_3(4-x) - \log_9(x-4)^2 = 2$		
	$2\log_3(4-x) - \frac{\log_3(x-4)^2}{\log_3 9} = 2$	**M1**	Change-base law
	$\log_3(4-x)^2 - \frac{\log_3(x-4)^2}{\log_3 3^2} = 2$	**M1**	Power law
	$\log_3(4-x)^2 - \frac{\log_3(x-4)^2}{2} = 2$		Identity property
	$2\log_3(4-x)^2 - \log_3(x-4)^2 = 4$ $\log_3(4-x)^4 - \log_3(x-4)^2 = 4$		
	$\log_3 \frac{(4-x)^4}{(x-4)^2} = 4$	**M1**	Quotient Law
	$\frac{(4-x)^4}{(x-4)^2} = 3^4$	**M1**	Convert Logarithmic form to Exponential form
	$(4-x)^4 = 81(x-4)^2$ $(x-4)^4 - 81(x-4)^2 = 0$ $(x-4)^2((x-4)^2 - 81) = 0$ $(x-4)^2 = 0$ or $(x-4)^2 = 81$ $x - 4 = 0$ or $x - 4 = \pm 9$		Solve factorised equation
	$x = 4$ or $x = -5$ or $x = 13$	**A1**	
	From (a), since $x < 4$, $\therefore x = -5$ is the only one real solution	**B1**	
	OR		
	$2\log_3(4-x) - \log_9(x-4)^2 = 2$		
	$2\log_3(4-x) - \frac{\log_3(x-4)^2}{\log_3 9} = 2$	**M1**	Change-base law
	$\log_3(4-x)^2 - \frac{\log_3(x-4)^2}{\log_3 3^2} = 2$	**M1**	Power law
	$\log_3(4-x)^2 - \frac{\log_3(x-4)^2}{2} = 2$		Identity property
	$\frac{\log_3(4-x)^2}{2} = 2$ $\log_3(4-x)^2 = 4$ $(4-x)^2 = 3^4$	**M1**	Convert Logarithmic form to

Qn	Solution	Marks	Testing
	$x^2-8x+16=81$		Exponential form
	$x^2-8x-65=0$	M1	
	$(x+5)(x-13)=0$		Writing $\log_3(\boldsymbol{x-4})$ is not correct as $4-x>0$, i.e. $x-4<0$
	$x=-5$ or $x=13$	A1	
	From (a), since $x<4$,		
	$\therefore x=-5$ is the only one real solution	B1	
	OR		
	$2\log_3(4-x)-\log_9(x-4)^2=2$	M1	Change-base law
	$2\log_3(4-x)-\dfrac{\log_3(x-4)^2}{\log_3 9}=2$		
	$\log_3(4-x)^2-\dfrac{\log_3(x-4)^2}{\log_3 3^2}=2$	M1	Power law
	$\log_3(4-x)^2-\dfrac{\log_3(x-4)^2}{2}=2$		Identity property
	$\log_3(4-x)^2-\log_3(4-x)=2$		
	$\log_3\dfrac{(4-x)^2}{(4-x)}=2$	M1	Quotient law
	$\dfrac{(4-x)^2}{(4-x)}=3^2$	M1	Convert Logarithmic form to Exponential form
	$x^2-8x+16=9(4-x)$		
	$x^2-8x+16=36-9x$		Writing $\log_3(\boldsymbol{x-4})$ is not correct as $4-x>0$, i.e. $x-4<0$
	$x^2+x-20=0$		
	$(x+5)(x-4)=0$		
	$x=-5$ or $x=4$	A1	
	From (a), since $x<4$,		
	$\therefore x=-5$ is the only one real solution	B1	
12	**Quadratic Equations**		
(a)	When $y=0$		
	$hx^2-2x-9h+6=0$		
	$hx^2-2x+(6-9h)=0$		
	For a quadratic curve,		
	coeff of $x^2\neq 0$,		
	$h\neq 0$ … [1]	M1	
	$D=(-2)^2-4h(6-9h)$	M1	$D=b^2-4ac$
	$=4-4h(6-9h)$		
	$=4(1-3h(2-3h))$		
	$=4(9h^2-6h+1)$		
	$=4(3h-1)^2$ or $(6h-2)^2$	M1	
	$D\geq 0$ …[2]		
	$\therefore$ from [1] & [2], $y=0$ has real roots for all values of h except $h=0$.		

Qn	Solution	Marks	Testing
(b)	For 2 real and equal roots,		
	$D=0$		
	$4(3h-1)^2=0$		
	$h=\mathbf{\dfrac{1}{3}}$	A1	
(c)	For line meeting the curve,		
	$hx^2-2x-9h+6=2x-h-12$		
	$hx^2-4x+18-8h=0$	M1	
	$D\geq 0$		
	$(-4)^2-4h(18-8h)\geq 0$	M1	
	$16-8h(9-4h)\geq 0$		
	$2-h(9-4h)\geq 0$		
	$4h^2-9h+2\geq 0$		
	$(4h-1)(h-2)\geq 0$	M1	
			
	$h\leq\frac{1}{4}$ or $h\geq 2$ ……[3]	A1	
	From [1] & [3],		
	$\boldsymbol{h\leq\frac{1}{4}}$ **or** $\boldsymbol{h\geq 2}$ **and** $\boldsymbol{h\neq 0}$	A1	
13	**Increasing / Decreasing Fns**		
(a)	$f(x)=x^2e^{(x+2)}$		
	$f'(x)=2xe^{(x+2)}+x^2e^{(x+2)}$	M1	Apply Product Rule
	$=xe^{(x+2)}(2+x)$		
	$=x(x+2)e^{(x+2)}$	A1	
	For decreasing function,		
	$f'(x)<0$		
	$x(x+2)e^{(x+2)}<0$	M1	Decreasing function
	since $e^{(x+2)}>0$		Must explain exponential function is positive for all values of x
	$x(x+2)<0$	M1	
			
	$\mathbf{-2<x<0}$		
	Maxima & Minima		
(b)	$f'(x)=(x^2+2x)e^{(x+2)}$		
	$f''(x)$ $=(2x+2)e^{(x+2)}+(x^2+2x)e^{(x+2)}$	M1	Apply Product Rule
	$f''(x)=(x^2+4x+2)e^{(x+2)}$		
	For the least gradient,		
	$f''(x)=0$		

Qn	Solution	Marks	Testing
	$(x^2+4x+2)e^{(x+2)}=0$	M1	
	$x^2+4x+2=0$		Quadratic formula or Completing the Square
	$x=\dfrac{-4\pm\sqrt{4^2-4(1)(2)}}{2(1)}$	M1	
	$=\dfrac{-4\pm\sqrt{8}}{2}$		
	$=\dfrac{-4\pm2\sqrt{2}}{2}$		
	$=-2\pm\sqrt{2}$	A1	
	Since the least gradient lies in $-2<x<0$,		
	$x=-2+\sqrt{2}$		
	Least gradient		
	$f'(x)=x(x+2)e^{(x+2)}$		
	$f'(-2+\sqrt{2})$		
	$=(-2+\sqrt{2})(-2+\sqrt{2}+2)e^{(-2+\sqrt{2}+2)}$		
	$=(-2+\sqrt{2})\sqrt{2}e^{\sqrt{2}}$		
	$=(2-2\sqrt{2})e^{\sqrt{2}}$	A1	

14 Remainder & Factor Thm

Qn	Solution	Marks	Testing
(a)	Given $f(x)=3x^3+2x^2+16$.		
	$f(3a)=f\left(-\frac{2}{3}\right)$		
	$3(3a)^3+2(3a)^2+16$	M1	Substitute values correctly
	$=3\left(-\frac{2}{3}\right)^3+2\left(-\frac{2}{3}\right)^2+16$		
	$3(27a^3)+2(9a^2)$		
	$=3\left(-\frac{8}{27}\right)+2\left(\frac{4}{9}\right)$		
	$81a^3+18a^2=-\frac{8}{9}+\frac{8}{9}$		
	$9a^2(9a+2)=0$	M1	Factorisation
	$\boldsymbol{a=0}$ **or** $\boldsymbol{a=-\frac{2}{9}}$	A1	
(b)	Subst $x=-2$ into $f(x)$,		
	$f(-2)=3(-2)^3+2(-2)^2+16$		Apply Factor Theorem
	$=-24+8+16$		
	$=0$	A1	
	$\therefore\ x=-2$ is a solution of $f(x)$.		

Qn	Solution	Marks	Testing
(c)	From (b),		
	$x+2$ is a factor of $f(x)$.		
	By inspection,		
	$f(x)=3x^3+2x^2+16$		
	$=(x+2)(3x^2+kx+8)$	M1	
	Equating coefficient of		
	x^2 : $k+6=2$, $k=-4$	M1	Comparing coefficients
	or		or
	x : $2k+8=0$, $k=-4$		Long Division
	$f(x)=3x^3+2x^2+16$		
	$=(x+2)(3x^2-4x+8)$	M1	Factorisation
	For $f(x)=0$		
	$(x+2)(3x^2-4x+8)=0$		
	$x+2=0$		
	$x=-2$		Solution for linear equation
	or $3x^2-4x+8=0$		
	$D=(4)^2-4(3)(8)$	M1	Use of discriminant or Quadratic formula
	$D=-80$		
	Since $D<0$, there is no real roots for the quadratic equation	B1	
	$\therefore f(x)=0$ has only one real root $x=-2$.		
(d)	$16y^3+2y-3=0$		
	$16+\frac{2}{y^2}-\frac{3}{y^3}=0$		
	$3\left(-\frac{1}{y}\right)^3+2\left(-\frac{1}{y}\right)^2+16=0$	M1	Transform into the required format
	Let $x=-\frac{1}{y}$		
	$3x^3+2x^2+16=0$		
	$x=-2$		
	$-\frac{1}{y}=-2$		
	$\boldsymbol{y=\frac{1}{2}}$	A1	

BEDOK SOUTH SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2025

4E5N

CANDIDATE NAME		CLASS		REGISTER NUMBER	

ADDITIONAL MATHEMATICS
Paper 2

4049 / 02
2 hours 15 minutes

90

Answer **all** the questions.

1 **(a)** Solve the equation $2(4^x)-8=11(2^x)-2^{2x+3}$, giving your answer correct to 3 significant figures. [5]

(b) Show that the solution from **part (a)** may be written in the form $m-\log_n p$ where m, n and p are integers to be determined. [2]

2 The mass, M grams, of a radioactive substance, present at time t years after first being observed, is given by the formula $M=200e^{-kt}$, where k is a constant. The mass of the substance was 123.7 g after being observed for 2 years.

(a) **(i)** State the initial mass of the substance. [1]

(ii) Show that k is approximately 0.240, correct to 3 significant figures. [1]

(iii) Find the mass of the substance when $t=5$, [1]

(iv) Find the value of t when the mass of the substance is 15% of its initial mass. Give your answers correct to three significant figures. [2]

(b) Explain, with clear working, why the mass of the substance can never be more than 200 grams. [1]

(c) Sketch the graph of M against t. [2]

3 **(a)** It is given that $y=x^2\sqrt{2x-5}$, where $x \geq h$.

Show that $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{kx(x-2)}{\sqrt{2x-5}}$, where k is an integer and determine the value of h and of k. [4]

(b) Hence evaluate $\int_3^7 \left\{\frac{10x(x-2)}{\sqrt{2x-5}}+6\right\}\mathrm{d}x$. [4]

4 **(a)** **(i)** Factorise completely x^3+64. [2]

(ii) **Hence**, express $\frac{3x^2}{x^3+64}$ in partial fractions. [5]

(b) Using the results in **part (a)**, or otherwise, find

$$\int \frac{2x^3+3x^2+128}{x^3+64}\,\mathrm{d}x.$$ [3]

5 A particle moves in a straight line so that, at time t seconds after passing a fixed point O, its velocity is v m/s, where $v = 4 + 8\cos 2t$. Find

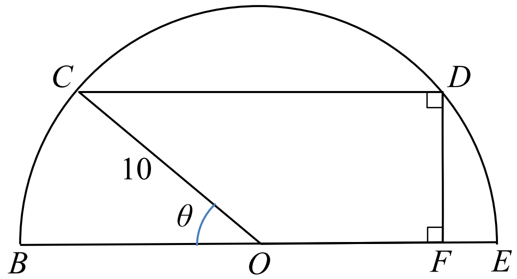
(a) the velocity of the particle at the instant it passes O, [1]

(b) the least value of the particle's acceleration, [1]

(c) the values of t, in terms of π, when the particle is at rest for $0 \le t \le 3$, [4]

(d) the distance travelled in the first 2 seconds. [4]

6



The diagram shows right angled trapezium $OCDF$ inside a semicircle with centre O and radius 10 cm such that angle BOC is θ radians, and angle CDF and angle OFD are right angles.

(a) Show that the perimeter, P cm, of trapezium $OCDF$ is given by

$$P = 10 + 30\cos\theta + 10\sin\theta$$ [2]

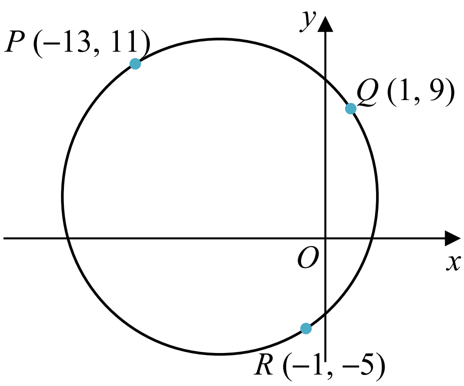
(b) Find the value of R when $10\sin\theta + 30\cos\theta$ is expressed as $R\cos(\theta - \alpha)$, where R and α are constants, and hence state the maximum perimeter of the trapezium. [3]

(c) Show that the area, A cm^2, of trapezium $OCDF$ is given by

$$A = 75\sin 2\theta$$ [2]

(d) The area of the trapezium varies with the value of θ. Find the value of θ for which the area has a stationary value and determine whether this area is a maximum or a minimum. [4]

7 **Solutions to this question by accurate drawing will not be accepted.**



In the diagram, P, Q and R are points on the circle.

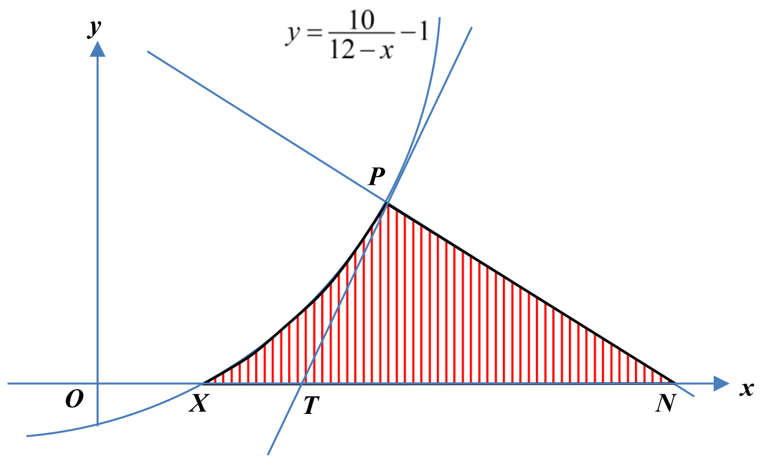
(a) Explain, with geometrical reason, why PR is the diameter of the circle. [3]

(b) Find the equation of the circle in the form $x^2 + y^2 + ax + by + c = 0$, where a, b and c are integers. [3]

(c) Find the equation of the perpendicular bisector of PQ. [3]

(d) The point S lies on the circle such that it is furthest from the point Q.
Show that the coordinates of S are $(-15, -3)$ and hence calculate the area of the quadrilateral $PQRS$. [3]

8



The diagram shows part of the curve $y = \frac{10}{12 - x} - 1$ passing through the point $P(2k, k - 1)$, where k is a constant. The curve meets the x-axis at the point X. The tangent and normal at P meet the x-axis at the points T and N respectively.

(a) Find the equation of the normal at P. [6]

(b) Find the **exact** area of the shaded region. [6]

9 The table below shows experimental values of two variables x and y.

x	1	2	3	4	5	6
y	2.20	1.74	1.71	1.83	1.87	1.96

The variables x and y are related by the equation $\frac{y}{a} = \frac{1}{x} + b\sqrt{x}$, where a and b are constants. One value of y has been recorded incorrectly.

(a) Show how $\frac{y}{a} = \frac{1}{x} + b\sqrt{x}$ is transformed to plot a graph of xy against $x\sqrt{x}$. [1]

(b) Draw a straight line graph of xy against $x\sqrt{x}$ for the given data. [2]

(c) Using your graph,

(i) find an approximate value of y to replace the incorrect value, [2]

(ii) estimate the value of a and of b, [3]

(iii) find the value of y when $x = 2.52$. [2]

(d) A pair of values of x and y is considered acceptable only if the xy value is within 2% vertical difference from the straight line. A student recorded a pair of values such that $x = 4.50$ and $y = 1.86$.
Verify whether the recorded values by the student are acceptable. [2]

-

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!\,r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\cos ec^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}.$$

Formulae for $\Delta\, ABC$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2} ab\sin C$$

ANSWERS
(Bedok South) 4E5N AM P2 (Prelim 2025) (w Ans)

1. (a) $x = 0.678$ (to 3 sig. fig.)

(b) $m = 3,\ n = 2,\ p = 5$

2. (a) **(i)** $M_0 = 200$ g

(ii) $k = 0.240$ (to 3 sig. fig.)

(iii) $M = 60.2$ g (to 3 sig. fig.)

(iv) $t = 7.90$ years (to 3 sig. fig.)

(c)

3. (a) $h = \frac{5}{2},\ k = 5$

(b) 300

4. (a) **(i)** $(x+4)(x^2 - 4x + 16)$

(ii) $\dfrac{1}{x+4} + \dfrac{2x-4}{x^2-4x+16}$

(b) $2x + \ln(x^3 + 64) + c$

5. (a) $v = 12$ m/s

(b) Least $a = -16$ m/s^2

(c) $t = \frac{1}{3}\pi,\ \frac{2}{3}\pi$

(d) 10.3 m (to 3 sig. fig.)

6. (a) **Hint:** Draw a line $CG \perp BE$

(b) $R = 10\sqrt{10}$ or 31.6 (to 3 sig. fig.)
Max $P = 41.6$ cm (to 3 sig. fig.)

(c) **Hint:** Apply Double Angle formula

(d) $\theta = \frac{\pi}{4}$

7. (a) **Hint:** Right angle in semicircle

(b) $x^2 + y^2 + 14x - 6y - 42 = 0$

(c) $y = 7x + 52$

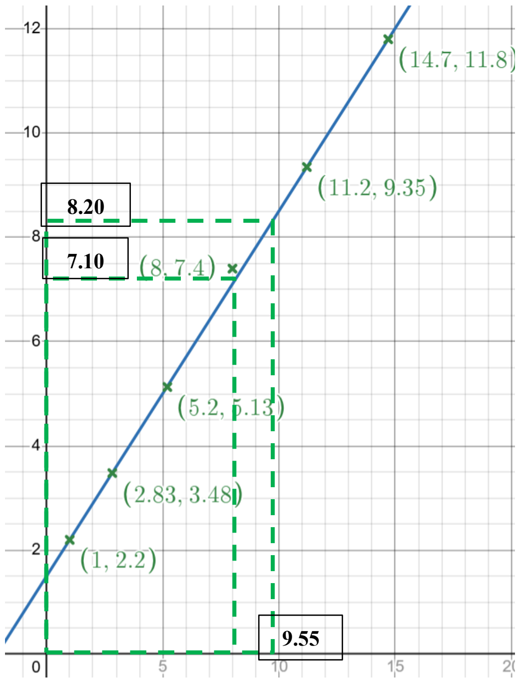
(d) **Hint:** The furthest two points on a circle is its diameter.
$Area\ of\ PQRS = 200$ units^2

8. (a) $y = -\frac{2}{5}x + 8$

(b) $10 \ln 5 + 12$

9. (a) **Hint:** Multiply equation with ax to get $xy = ab\left(x\sqrt{x}\right) + a$

(b)



(c) **(i)** $y = 1.78$ (to 3 sig. fig.)

(ii) $a = 1.50$ (to 3 sig. fig.)

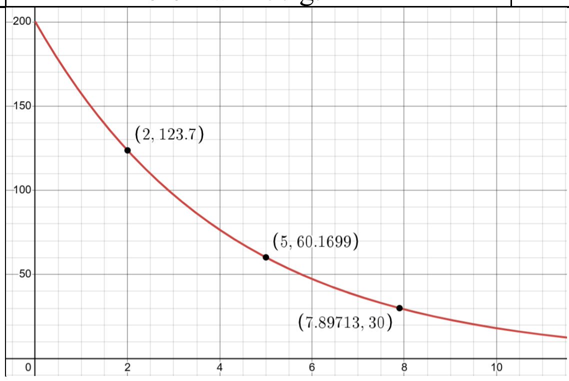
$b = 0.467$ (to 3 sig. fig.)

(iii) $y = 1.71$ (to 3 sig. fig.)

(d) Unacceptable,
Vertical difference = 2.073%

MARKING SCHEME

(Bedok South) 4E5N AM P2 (Prelim 2023) (w Ans)

Qn	Solution	Testing
1.	**Exponential Equations**	
(a)	$2(4^x) - 8 = 11(2^x) - 2^{2x+3}$ $2(2^{2x}) - 8 = 11(2^x) - (2^{2x})2^3$ $2(2^x)^2 - 8 = 11(2^x) - 8(2^x)^2$	Convert to base 2
	Let $u = 2^x$ $2u^2 - 8 = 11u - 8u^2$ $10u^2 - 11u - 8 = 0$ $(2u+1)(5u-8) = 0$	Apply substitution correctly
	$u = -\frac{1}{2}$ or $u = \frac{8}{5}$ $2^x = -\frac{1}{2}$ (reject) $2^x = \frac{8}{5}$	Solving for u
	$\lg 2^x = \lg \frac{8}{5}$ $x = \frac{\lg \frac{8}{5}}{\lg 2}$ $= 0.678072$ $= \mathbf{0.678}$ (to 3 sig. fig.)	Apply logarithm appropriately
(b)	From (a), $2^x = \frac{8}{5}$ $\log_2 2^x = \log_2 \frac{8}{5}$ $x \log_2 2 = \log_2 8 - \log_2 5$	Apply $\log_2$ onto equation
	$x = \log_2 2^3 - \log_2 5$ $= 3\log_2 2 - \log_2 5$	Power law Quotient law
	$= 3 - \log_2 5$ $= m - \log_n p$ $m = \mathbf{3}$, $n = \mathbf{2}$, $p = \mathbf{5}$	Identity property
2.	**Exponential Functions (Word Problems)**	
(a) (i)	When $t = 0$, $M = 200e^{-kt}$ $M_0 = 200e^{-k(0)}$ $= \mathbf{200\ g}$	
(a) (ii)	When $t = 2$, $M = 123.7$ $M = 200e^{-kt}$ $123.7 = 200e^{-k(2)}$ $\frac{123.7}{200} = e^{-2k}$ $\ln\left(\frac{123.7}{200}\right) = -2k$	Take natural logarithm on both sides of equation
	$k = -\frac{1}{2}\ln\left(\frac{123.7}{200}\right)$ $= 0.240229$ $= \mathbf{0.240}$ (to 3 sig. fig.)	
(a) (iii)	When $t = 5$, For $k = 0.24$: $M = 200e^{-kt}$, $= 200e^{-0.24(5)}$, $= 60.23884$ For $k = 0.240229$: $M = 200e^{-kt}$, $= 200e^{-0.240229(5)}$, $= 60.16991$ $= \mathbf{60.2\ g}$ (to 3 sig. fig.)	
(a) (iv)	$M = M_0 e^{-kt}$ $\frac{M}{M_0} = e^{-kt}$ $15\% = e^{-kt}$ $0.15 = e^{-kt}$ For $k = 0.24$: $0.15 = e^{-0.24t}$, $\ln 0.15 = -0.24t$, $t = -\frac{\ln 0.15}{0.24}$, $= 7.90467$ For $k = 0.240229$: $0.15 = e^{-0.240229t}$, $\ln 0.15 = -0.240229t$, $t = -\frac{\ln 0.15}{0.240229}$, $= 7.89713$ $t = \mathbf{7.90\ years}$ (to 3 sig. fig.)	Take natural logarithm on both sides of equation
(b)	Since for $t \geq 0$, $e^{kt} \geq 1$ $\frac{1}{e^{kt}} \leq 1$ $e^{-kt} \leq 1$ $200e^{-kt} \leq 200$ $M \leq 200$ $\therefore$ mass of substance can never be more than 200 g.	
(c)		

G1 – Initial state (0, 200)
G1 – Smooth curve decreasing towards asymptote t-axis

Qn	Solution	Testing
3.	**Differentiation (Product Rule)**	
(a)	For square root to be defined,	
	$2x-5\geq 0 \quad \Rightarrow \quad x\geq \frac{5}{2}$	Square root criterion
	$\therefore\ \boldsymbol{h=\frac{5}{2}}$	
	$y=x^2\sqrt{2x-5}$	
	$y=x^2\,(2x-5)^{\frac{1}{2}}$	
	$\frac{\mathrm{d}y}{\mathrm{d}x}=2x(2x-5)^{\frac{1}{2}}$	
	$+x^2(2)\frac{1}{2}(2x-5)^{-\frac{1}{2}}$	Apply Product rule
	$\frac{\mathrm{d}y}{\mathrm{d}x}=2x\sqrt{2x-5}+\frac{x^2}{\sqrt{2x-5}}$	
	$=\frac{2x(2x-5)+x^2}{\sqrt{2x-5}}$	Combine to single fraction
	$=\frac{2x(2x-5)+x^2}{\sqrt{2x-5}}$	
	$=\frac{5x^2-10x}{\sqrt{2x-5}}$	
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{5x(x-2)}{\sqrt{2x-5}}$	
	$\therefore\ \boldsymbol{k=5}$	
	Integration (Reverse of Differentiation)	
(b)	$\int_3^7\left\{\frac{10x(x-2)}{\sqrt{2x-5}}+6\right\}\mathrm{d}x$	
	$=2\int_3^7\frac{5x(x-2)}{\sqrt{2x-5}}\mathrm{d}x+\int_3^7 6\,\mathrm{d}x$	Split into 2 integrals
	$=2\left[x^2\sqrt{2x-5}\right]_3^7+6[x]_3^7$	Integration as reversed differentiation
	$=2\left\{7^2\sqrt{2(7)-5}-3^2\sqrt{2(3)-5}\right\}$	
	$+6(7-3)$	Substitution of boundaries
	$=2\{49\sqrt{9}-9\sqrt{1}\}+24$	
	$=276+24$	
	$\boldsymbol{=300}$	
	OR	
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{5x(x-2)}{\sqrt{2x-5}}$	
	$\int_3^7\frac{5x(x-2)}{\sqrt{2x-5}}\mathrm{d}x=\left[x^2\sqrt{2x-5}\right]_3^7$	Integration as reversed differentiation

Qn	Solution	Testing
	$\int_3^7\frac{10x(x-2)}{\sqrt{2x-5}}\mathrm{d}x=2\left[x^2\sqrt{2x-5}\right]_3^7$	
	$\int_3^7\frac{10x(x-2)}{\sqrt{2x-5}}\mathrm{d}x+\int_3^7 6\,\mathrm{d}x$	Transform to required integral
	$=2\left\{7^2\sqrt{2(7)-5}-3^2\sqrt{2(3)-5}\right\}$	
	$+\int_3^7 6\,\mathrm{d}x$	Substitution of boundaries
	$\int_3^7\left\{\frac{10x(x-2)}{\sqrt{2x-5}}+6\right\}\mathrm{d}x$	
	$=2\{49\sqrt{9}-9\sqrt{1}\}+6[x]_3^7$	
	$=276+6(7-3)$	
	$\boldsymbol{=300}$	
4.	**Cubic Identities**	
(a) (i)	x^3+64	
	$=(x)^3+(4)^3$	Apply sum of cubes identity
	$=(x+4)((x)^2-(x)(4)+(4)^2)$	
	$\boldsymbol{=(x+4)(x^2-4x+16)}$	
	Partial Fractions	
(a) (ii)	$\frac{3x^2}{x^3+64}$	
	$=\frac{3x^2}{(x+4)(x^2-4x+16)}$	Express proper fraction in partial fractions with unknown coefficients
	$=\frac{A}{x+4}+\frac{Bx+C}{x^2-4x+16}$	
	$A(x^2-4x+16)$	
	$+(Bx+C)(x+4)=3x^2$	
	Let $x=-4$,	Use Substitution and/or Equating coefficients methods to find unknowns
	$A((-4)^2-4(-4)+16)$	
	$=3(-4)^2$	
	$A(16+16+16)=3(16)$	
	$A=1$	
	Let $x=0$,	
	$A(16)+C(4)=0$	Award 1 mark for at least 2 correct values
	$1(16)+4C=0$	
	$16+4C=0$	
	$C=-4$	
	Let $x=1$,	
	$A(1-4+16)$	
	$+(B+C)(1+4)=3$	
	$1(13)+(B-4)(5)=3$	
	$13+5B-20=3$	

Qn	Solution	Testing
	$B = 2$	
	OR Equating coefficient of x^2 : $A + B = 3$, $B = 3 - A \cdots (1)$ x : $-4A + 4B + C = 0 \cdots (2)$ x^0 : $16A + 4C = 0$, $C = -4A \cdots (3)$ Subst (1) & (3) into (2), $-4A + 4(3 - A) + (-4A) = 0$ $-4A + 12 - 4A - 4A = 0$ $12A = 12$ $A = 1$ $B = 3 - 1 = 2$ $C = -4(1) = -4$	
	$\therefore \frac{3x^2}{x^3+64} = \mathbf{\frac{1}{x+4} + \frac{2x-4}{x^2-4x+16}}$	
	Indefinite Integral	
(b)	$\int \frac{2x^3+3x^2+128}{x^3+64} \mathrm{d}x$ $= \int \frac{2(x^3+64)}{x^3+64} \mathrm{d}x + \int \frac{3x^2}{x^3+64} \mathrm{d}x$ $= \int 2 \, \mathrm{d}x + \int \frac{3x^2}{x^3+64} \mathrm{d}x$ $= \mathbf{2x + \ln(x^3+64) + c}$	Split into 2 integrals Correct integration
5.	**Kinematics (Trigonometric Functions)**	
(a)	When particle passes O, $t = 0$, Hence, $v = 4 + 8\cos 2(0)$ $= \mathbf{12 \ m/s}$	
(b)	$v = 4 + 8\cos 2t$ $\frac{\mathrm{d}v}{\mathrm{d}t} = -(2)8\sin 2t$ $a = -16\sin 2t$ Least acceleration $= \mathbf{-16 \ m/s^2}$	
(c)	$0 \le t \le 3$ $0 \le 2t \le 6$ When particle is at rest, $v = 0$ m/s,	

Qn	Solution	Testing
	$4 + 8\cos 2t = 0$ $\cos 2t = -\frac{1}{2}$ Basic angle $\alpha = \cos^{-1}\left(\frac{1}{2}\right) = \frac{1}{3}\pi$ For $\cos 2t < 0$, $2t$ lies in Q2, Q3 $2t = \pi - \frac{1}{3}\pi, \ \pi + \frac{1}{3}\pi$ $= \frac{2}{3}\pi, \ \frac{4}{3}\pi$ $t = \frac{1}{3}\pi, \ \frac{2}{3}\pi$	Correct Substitution Basic Angle Angles in correct quadrants
(d)	$s = \int_0^{\frac{\pi}{3}} (4 + 8\cos 2t) \, \mathrm{d}t$ $s = 4t + 4\sin 2t + c$ (table below) Distance travelled in the first 2 sec $= (7.65289 - 0)$ $+(7.65289 - 4.97279)$ $= 10.33299$ $= \mathbf{10.3 \ m}$ (to 3 sig. fig.)	Correct Integration
(d)	Distance travelled in the first 2 sec $= \left\| \int_0^{\frac{\pi}{3}} (4 + 8\cos 2t) \, \mathrm{d}t \right\|$ $+ \left\| \int_{\frac{\pi}{3}}^{2} (4 + 8\cos 2t) \, \mathrm{d}t \right\|$ $= \left\| [4t + 4\sin 2t]_0^{\frac{\pi}{3}} \right\|$ $+ \left\| [4t + 4\sin 2t]_{\frac{\pi}{3}}^{2} \right\|$ $= \left\| 4\left(\frac{\pi}{3}\right) + 4\left(\sin\frac{2\pi}{3}\right) \right\|$ $+ \left\| 4\left(2 - \frac{\pi}{3}\right) + 4\left(\sin 4 - \sin\frac{2\pi}{3}\right) \right\|$ $= 7.65289 + 2.6801$ $= 10.33299$ $= \mathbf{10.3 \ m}$ (to 3 sig. fig.)	Correct Expressions Correct Integration Correct Substitution

Table in 5(d):

t	s
0	0
$\frac{1}{3}\pi$	7.65289
2	4.97279

Qn	Solution	Testing
6.	**R-Formulae**	
(a)	Draw a line $CG \perp BE$. In ΔCGO, $\angle CGO = \frac{\pi}{2}$, $OC = 10$ $CG = 10\sin\theta$ $GO = 10\cos\theta$ $DF = CG = 10\sin\theta$ $CD = GF = 2GO$ $= 20\cos\theta$ $FO = GO = 10\cos\theta$ $P = OC + CD + DF + FO$ $= 10 + 20\cos\theta + 10\sin\theta + 10\cos\theta$ $P = 10 + 30\cos\theta + 10\sin\theta$	
(b)	$30\cos\theta + 10\sin\theta$ $= R\cos(\theta - \alpha)$ $= \sqrt{30^2 + 10^2}\cos(\theta - \alpha)$ $= 10\sqrt{10}\cos(\theta - \alpha)$ where $\mathbf{R = 10\sqrt{10}}$ $= 31.6227$ $\mathbf{= 31.6}$ (to 3 s.f.) $Max\ P = 10 + R$ $\mathbf{= 41.6}$ **cm** (to 3 s.f.)	
(c)	$A = \frac{1}{2}DF(CD + FO)$ $= \frac{1}{2}(10\sin\theta)(20\cos\theta + 10\cos\theta)$ $A = 5\sin\theta\,(30\cos\theta)$ $A = 150\sin\theta\cos\theta$ $A = 75\sin 2\theta$	Area $= \frac{1}{2} \times base \times ht$ Double angle formula
	Stationary Values	
(d)	$\frac{dA}{d\theta} = 150\cos 2\theta$ $\frac{d^2A}{d\theta^2} = -300\sin 2\theta$ For stationary value, $\frac{dA}{d\theta} = 0$ $150\cos 2\theta = 0$ $2\theta = \frac{\pi}{2}$ $\theta = \frac{\pi}{4}$ $\frac{d^2A}{d\theta^2} = -300\sin 2\theta$ $= -300\sin\frac{\pi}{2}$ $= -300$ $\frac{d^2A}{d\theta^2} < 0$ $A = 75\sin 2\theta$ $= 75\sin\frac{\pi}{2}$ $= 75$ A has a maximum area (of 75 cm^2) when $\theta = \frac{\pi}{4}$.	Value of A is not required
7.	**Circles & Coordinate Geometry**	
(a)	$P(-13, 11)$ $Q(\ 1,\ 9)$ $R(-1, -5)$ Gradient of PQ: $m_{PQ} = \frac{11-9}{-13-1} = -\frac{1}{7}$ Gradient of QR: $m_{QR} = \frac{9-(-5)}{1-(-1)} = 7$ Since $m_{PQ} \times m_{QR} = -\frac{1}{7} \times 7 = -1$ $\Rightarrow PQ$ is perpendicular to QR $\Rightarrow \angle PQR = 90°$ (**right angle in semicircle**) $\Rightarrow PR$ is the diameter of circle.	Applying $m_1 \times m_2 = -1$
	OR $PQ^2 = (-13-1)^2 + (11-9)^2$ $= 200$ $QR^2 = (1-(-1))^2 + (9-(-5))^2$ $= 200$ $PR^2 = (-13+1)^2 + (11+5)^2$ $= 400$ Since $PR^2 = PQ^2 + QR^2 = 400$, by converse of Pythagoras' Theorem, $\angle PQR = 90°$. (**right angle in semicircle**) $\Rightarrow PR$ is the diameter of circle.	

Qn	Solution	Testing
	OR Midpoint of PR, $M = \left(\frac{-13-1}{2}, \frac{11-5}{2}\right)$ $= (-7,\ 3) \ldots [1]$ $PM = \sqrt{(-13+1)^2 + (11-3)^2}$ $= 10$ $QM = \sqrt{(1+7)^2 + (9-3)^2}$ $= 10$ $RM = (-1+7)^2 + (-5-3)^2 = 10$ $\therefore PM = QM = RM = 10 \ldots [2]$ From [1] and [2], **midpoint of chord PR is equidistant to P, Q & R** $\Rightarrow PR$ is the diameter of circle.	
(b)	As centre is midpoint of diameter, $\text{centre} = \left(\frac{-13-1}{2}, \frac{11-5}{2}\right)$ $= (-7, 3)$ $\text{radius} = \frac{1}{2}PR$ $= \frac{1}{2}\sqrt{(-13-(-1))^2 + (11-(-5))^2}$ $= \frac{1}{2}\sqrt{400}$ $= 10$ Equation of circle $(x-(-7))^2 + (y-3)^2 = 10^2$ $x^2 + 14x + 49 + y^2 - 6y + 9 = 100$ $\mathbf{x^2 + y^2 + 14x - 6y - 42 = 0}$	Award 1 mark for either centre or radius correct Correct substitution
(c)	Let $T(x, y)$ be a general point on the perpendicular bisector of PQ. $PT = QT$ $\sqrt{(x-(-13))^2 + (y-11)^2}$ $= \sqrt{(x-1)^2 + (y-9)^2}$ $(x+13)^2 + (y-11)^2$ $= (x-1)^2 + (y-9)^2$ $x^2 + 26x + 169 + y^2 - 22y + 121$ $= x^2 - 2x + 1 + y^2 - 18y + 81$ $4y = 28x + 208$ $\mathbf{y = 7x + 52}$	
	OR Perpendicular bisector passes through the centre. Using centre $(-7,\ 3)$ and $m_{\perp PQ} = 7$	

Qn	Solution	Testing
	Equation of perpendicular bisector of QR is $y - 3 = 7(x - (-7))$ $\mathbf{y = 7x + 52}$	
	OR Perpendicular bisector passes through the midpoint of chord. Midpoint of PQ $= \left(\frac{-13+1}{2}, \frac{11+9}{2}\right)$ $= (-6,\ 10)$ $m_{\perp PQ} = 7$ Equation of perpendicular bisector of QR is $y - 10 = 7(x - (-6))$ $\mathbf{y = 7x + 52}$	
(d)	The furthest two points on a circle is the diameter of the circle. $\therefore$ centre $(-7, 3)$ is midpoint of diameter QS, $\frac{x_Q + x_S}{2} = -7$; $\frac{y_Q + y_S}{2} = 3$ $1 + x_S = -14$; $9 + y_S = 6$ $x_S = -15$; $y_S = -3$ $\therefore \mathbf{S(-15, -3)}$	Diameter is the longest chord Mid-point
	Area of quadrilateral $PQRS$ $= \frac{1}{2}\begin{vmatrix} P & Q & R & S & P \\ -13 & 1 & -1 & -15 & -13 \\ 11 & 9 & -5 & -3 & 11 \end{vmatrix}$ $= \frac{1}{2}\begin{vmatrix} (-117 - 5 + 3 - 165) \\ -(11 - 9 + 75 + 39) \end{vmatrix}$ $\mathbf{= 200\ \text{units}^2}$	Shoelace method
8.	**Tangent & Normal**	
(a)	$y = \frac{10}{12 - x} - 1 \ldots\ldots [1]$ Subst $P(2k, 4)$ into [1] $4 = \frac{10}{12 - 2k} - 1$ $5 = \frac{5}{6 - k}$ $6 - k = 1$ $k = 5$ $P(10, 4)$ $\frac{dy}{dx} = (-10)(12 - x)^{-2}(-1)$ $\frac{dy}{dx} = \frac{10}{(12 - x)^2} \ldots\ldots [2]$	Chain rule

Qn	Solution	Testing

Subst $P(10, 4)$ into [2]

$$\frac{dy}{dx} = \frac{10}{(12-10)^2} = \frac{5}{2}$$

$$m_{PT} = \frac{5}{2} \text{ and } m_{PN} = -\frac{2}{5}$$

Using $P(10, 4)$ and $m_{PN} = -\frac{2}{5}$,

Equation of normal at P

$$y - 4 = -\frac{2}{5}(x - 10)$$

$$y - 4 = -\frac{2}{5}x + 4$$

$$\boldsymbol{y = -\frac{2}{5}x + 8} \quad \ldots[3]$$

Area under Curve

(b) Subst $X(x_X, 0)$ into [1]

$$0 = \frac{10}{12 - x_X} - 1$$

$$\frac{10}{12 - x_X} = 1$$

$$10 = 12 - x_X$$

$$x_X = 2$$

Subst $N(x_N, 0)$ into [3]

$$0 = -\frac{2}{5}x_N + 8$$

$$\frac{2}{5}x_N = 8$$

$$x_N = 20$$

Area of shaded region

$$\int_2^{10}\left(\frac{10}{12-x} - 1\right)dx + \frac{1}{2}(20-10)(4)$$

$$= [-10\ln(12-x) - x]_2^{10} + 20$$ — Integration

$$= -[10\ln(12-x) + x]_2^{10} + 20$$

$$= [10\ln(12-x) + x]_{10}^{2} + 20$$

$$= 10\ln\left(\frac{12-2}{12-10}\right) + \{2 - 10\} + 20$$ — Substitution of boundaries

$$= 10\ln 5 + 12$$

OR

Area of shaded region

$$\int_2^{10}\left(\frac{10}{12-x} - 1\right)dx + \int_{10}^{20}\left(-\frac{2}{5}x + 8\right)dx$$

$$= [-10\ln(12-x) - x]_2^{10} + \left[-\frac{1}{5}x^2 + 8x\right]_{10}^{20}$$ — Integration

$$= -10\ln\left(\frac{12-10}{12-2}\right) - \{10 - 2\} - \frac{1}{5}\{20^2 - 10^2\} + 8\{20 - 10\}$$ — Substitution of boundaries

$$= 10\ln 5 + 12$$

9. Linear Law

(a)

$$\frac{y}{a} = \frac{1}{x} + b\sqrt{x}$$

Multiply equation with ax,

$$xy = a + abx\sqrt{x}$$

$$xy = ab\left(x\sqrt{x}\right) + a$$

Linearise equation $Y = mX + C$

(b)

x	1	2	3	4	5	6
y	2.20	1.74	1.71	1.85	1.87	1.96
$x\sqrt{x}$	1.00	2.83	5.20	8.00	11.2	14.7
xy	2.20	3.48	5.13	7.40	9.35	11.8

Tabulate values for Y vs. X

G1 – Plotted points
G1 – Best-fit line

(c)(i) From the graph,
the incorrect point is (8, 7.40).

Corresponding to $x = 4$ (in the table),
For $x\sqrt{x} = 8$
Correct $xy = 7.05 \ to \ 7.15$

$$4y = 7.05 \ to \ 7.15$$

$$y = 1.7625 \ to \ 1.7875$$

$$y = \mathbf{1.76} \ to \ \mathbf{1.79}$$

(to 3 sig. fig.) — Read from graph

(c)(ii) From the graph,

xy intercept $= 1.5$

$$a = \mathbf{1.50} \text{ (to 3 sig. fig.)}$$ — Read from graph

$$\text{Gradient} = \frac{11.8 - 2.20}{14.7 - 1.00}$$

$$ab = 0.7007$$

$$1.50\,b = 0.7007$$

$$b = 0.467153$$

$$b = \mathbf{0.467} \text{ (to 3 sig. fig.)}$$

(c)(iii) When $x = 2.52$,
for $x\sqrt{x} = 2.52\sqrt{2.52}$
$= 4.00037$
From the graph,

for $x\sqrt{x} = 4.00$

$$xy = 4.3$$ — Read from graph

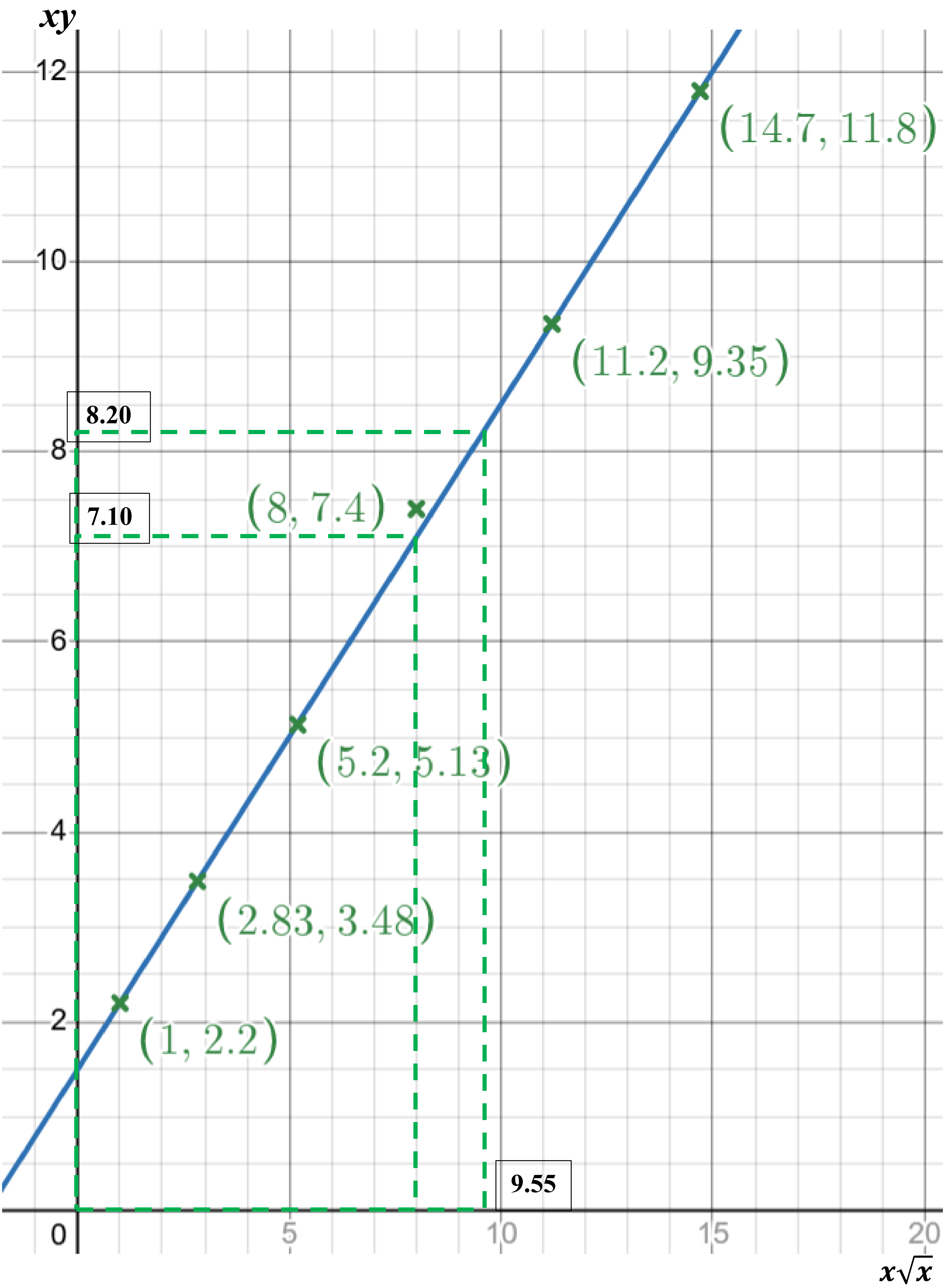
$$2.52\,y = 4.3$$

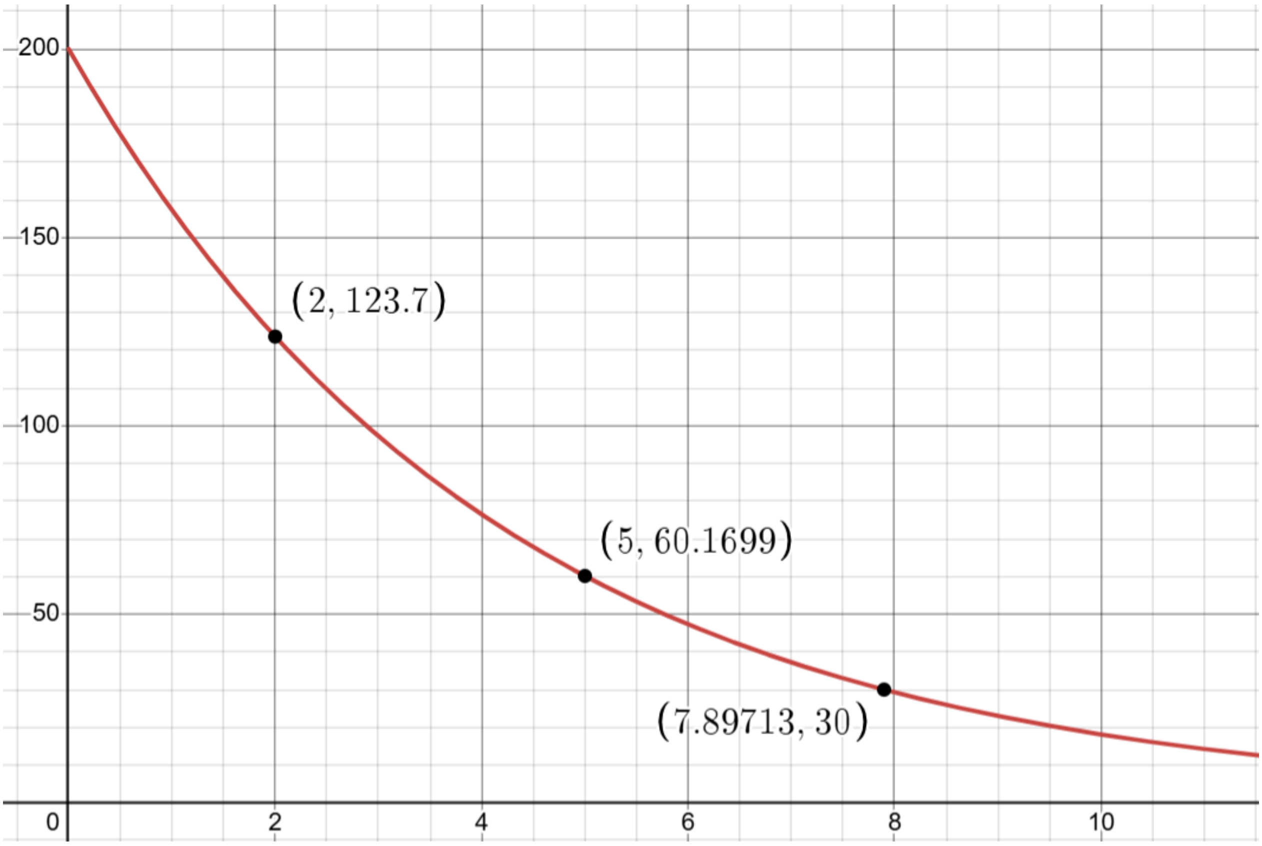
$$y = 1.70634$$

$$y = \mathbf{1.71} \text{ (to 3 sig. fig.)}$$

Qn	Solution	Testing
(d)	From readings of experiment, $x = 4.50$ and $y = 1.86.$ $x\sqrt{x} = 9.5459$ $xy = (4.50)(1.86)$ $= 8.37$ From the graph, for $x\sqrt{x} = 9.5459$ $xy = 8.20$ Vertical difference $= \frac{8.37 - 8.20}{8.20} \times 100\%$ $= 2.073\% > 2\%$ Since vertical difference is **more than 2%**, the experimental readings are **unacceptable**.	Read from graph

Qn	Solution	Testing
	OR Acceptable range $= 8.20 \times (100 \pm 2)\%$ $= 8.036\ \ to\ \ 8.364$ Since $xy = 8.37$ is **out of** the acceptable **range**, the experimental readings are **unacceptable**.	


xy
12
10
8
6
4
2
0
5
10
15
20
x√x
8.20
7.10
9.55
(14.7, 11.8)
(11.2, 9.35)
(8, 7.4)
(5.2, 5.13)
(2.83, 3.48)
(1, 2.2)


200
150
100
50
0
2
4
6
8
10
(2, 123.7)
(5, 60.1699)
(7.89713, 30)

AHMAD IBRAHIM SECONDARY SCHOOL
GCE O-LEVEL PRELIMINARY EXAMINATION 2025

SECONDARY 4 EXPRESS

Name:	Class:	Register No.:

ADDITIONAL MATHEMATICS **4049/01**
Paper 1 **12 August 2025**

Candidates answer on the Question Paper. **2 hours 15 minutes**

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.
Give non-exact numerical answers to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 90.

For Examiner's Use
/90

This document consists of **19** printed pages.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2\sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$\Delta = \frac{1}{2}bc\sin A$$

1 It is given that P, Q and R are the angles of a triangle.

(a) Show that $\cos P = -\cos(Q + R)$. [2]

(b) Given that $Q = 45°$ and $R = 60°$, find $\cos P$ in the form $\frac{1}{4}(\sqrt{a} - \sqrt{b})$, where a and b are integers. [3]

2 Baking powder is poured onto a flat surface at a constant rate of 2π cm^3s^{-1} and formed a right circular cone. The radius of the cone is always $\frac{1}{18}$ of its height. Find the rate of change of the radius of the cone after 3 seconds of pouring. [5]

3 **(a)** Determine the set of values of m for which the equation $2x^2 + 4x + 2m = 6mx - 2$ has real roots. [4]

(b) Hence state what can be deduced about the curve $y = 2(x+1)^2$ and the line $y = 6x - 2$. Justify your statement. [2]

4 **(a)** Show that $\frac{\mathrm{d}}{\mathrm{d}x}\left(\ln\left(\cos x\right)\right) = -\tan x$. [2]

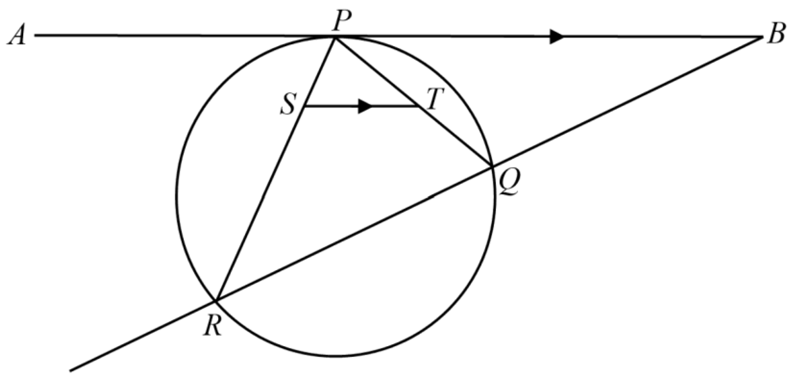
(b) Differentiate $x\tan x$ with respect to x. [2]

(c) Using the results from part **(a)** and **(b)**, find $\int x\sec^2 x\ \mathrm{d}x$ and hence show that $\int_0^{\frac{\pi}{4}} x\sec^2 x\ \mathrm{d}x = \frac{\pi}{4} - \frac{1}{2}\ln 2$. [4]

5 **(a)** In the expansion of $(2+x)^n$, where n is a positive integer, the coefficient of x^2 is twice the coefficient of x. Find the value of n. [3]

(b) Find the value of the term that is independent of x in the expansion of $\left(2x-\frac{1}{4x^4}\right)^{15}$. [4]

6



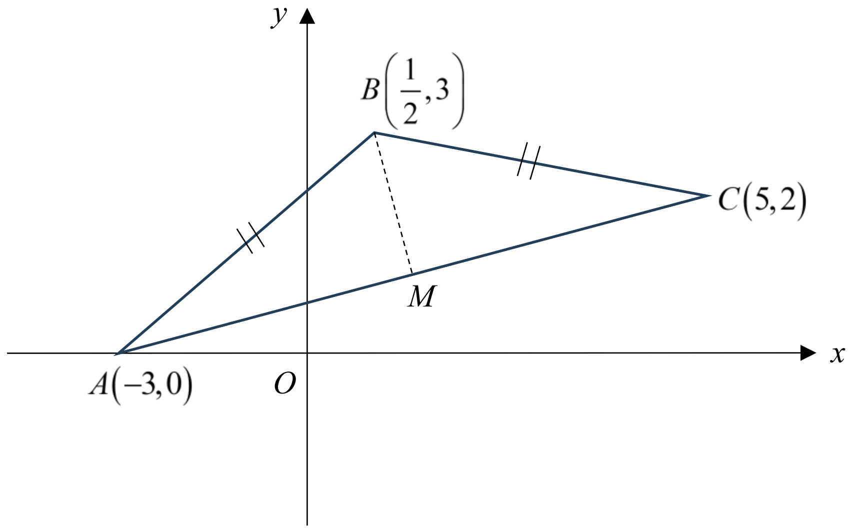
The diagram shows a circle passing through the points P, Q and R. The point Q lies on the line RB. AB is a tangent to the circle at P. The points S and T lie on PR and PQ respectively. Given that AB is parallel to ST, prove that

(a) triangle PST is similar to triangle PQR, [3]

(b) $PQ \times PT = PR \times PS$, [2]

(c) Determine if *STQR* is a cyclic quadrilateral. [4]

7



The diagram shows an isosceles triangle *ABC* in which $A(-3,0)$, $B\left(\frac{1}{2},3\right)$ and $C(5,2)$. *M* is the foot of perpendicular from *B* to *AC*.

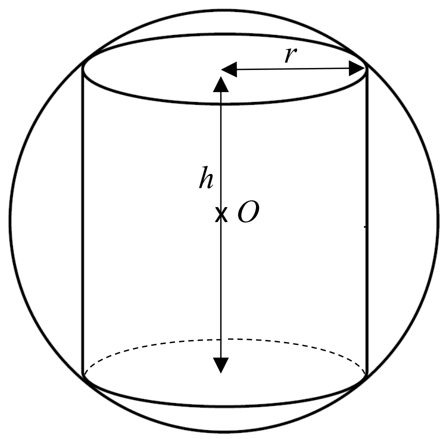
(a) Find the coordinates of *M*. [1]

(b) Find the equation of the perpendicular bisector of *AC*. [2]

(c) Given that *ABCD* is a kite with $BM = \frac{2}{7}BD$, find the coordinates of *D*. [3]

(d) Find the area of the kite *ABCD*. [2]

8



A prototype consists of a cylindrical container of height h cm and radius r cm inscribed in a hollow sphere with centre O.

The sphere has a surface area of 6400π cm^2 and both the sphere and container have negligible thickness.

(a) Show that the volume of the cylinder container, V cm^3, is given by [3]

$$V = 2\pi r^2\sqrt{1600 - r^2}\ .$$

(b) Given that r can vary, find the value of r for which the volume V is stationary. [5]

(c) A scientist plans to launch this prototype into outer space carrying as much fuel as possible. Explain whether the prototype can satisfy the scientist's requirement. [2]

9 It is given that $f(x) = 4 + \cos\left(\frac{x}{2}\right)$ and $g(x) = -2\sin x$.

(a) State the period and amplitude of f(x). [2]

(b) State the period and amplitude of g(x). [1]

(c) Sketch, on the same axes, the graphs of y = f(x) and y = g(x) for $0° \leq x \leq 360°$. [3]

10 **(a)** Express $\frac{1-3x-3x^2}{x(x+1)^2}$ in partial fractions. [5]

(b) Hence find $\int \frac{1-3x-3x^2}{2x(x+1)^2}\,dx$. [4]

11

The horizontal distance of a child on a carousel, h m, from the starting point is modelled by the equation, $h = 2(1 - \cos kt)$, where k is a constant and t is the time in seconds after the child leaves the starting point. The time to complete one revolution is 20 seconds.

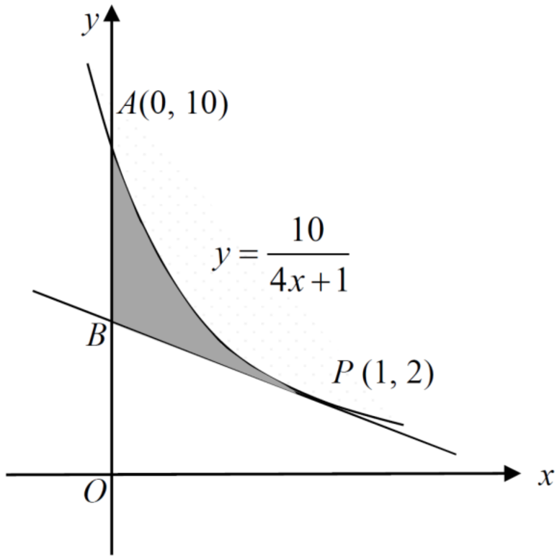
(a) Explain why this model suggests that the diameter of the carousel is 4 m. [1]

(b) Show that the value of k is $\frac{\pi}{10}$ radians per second. [2]

(c) As the carousel turns, it is possible for the child on the carousel to view a landmark, provided that the horizontal distance of the child is within 1 m from the starting point.

Find the duration of time for which the child will not be able to view the landmark during one revolution. [5]

12



The diagram shows part of the curve $y = \frac{10}{4x+1}$ intersecting the y-axis at $A(0, 10)$. The tangent to the curve at the point $P(1, 2)$ intersects the y-axis at B.

(a) Show that the coordinates of B is $(0, 3.6)$. [4]

(b) Find the **exact** area of the shaded region. [5]

END OF PAPER

AHMAD IBRAHIM SECONDARY SCHOOL
GCE O-LEVEL PRELIMINARY EXAMINATION 2025

SECONDARY 4 EXPRESS

Name:	Class:	Register No.:

ADDITIONAL MATHEMATICS **4049/01**
Paper 2 **13 August 2025**

Candidates answer on the Question Paper. **2 hours 15 minutes**

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.
Give non-exact numerical answers to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 90.

For Examiner's Use
/90

This document consists of **19** printed pages.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$\Delta = \frac{1}{2}bc\sin A$$

1 A curve has the equation $y = \dfrac{\sin 2x}{2 - \cos 2x}$.

(a) Show that the gradient function can be expressed in the form $\dfrac{k\cos 2x - 2}{(2 - \cos 2x)^2}$, where k is a constant. [3]

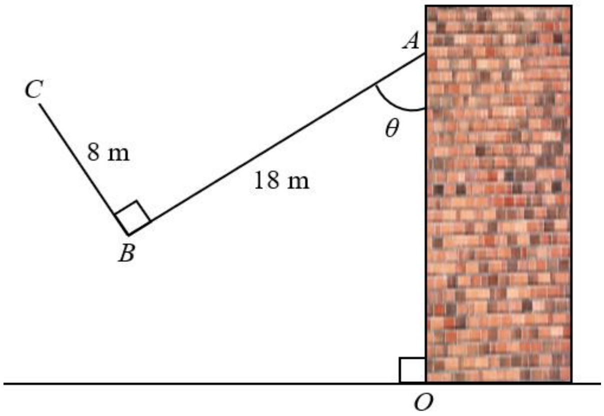
(b) Find the acute angle between the tangent to the curve at $x = \dfrac{\pi}{12}$ and the line $y = 0$. [3]

2 **(a)** Factorise x^3+27k^3 as a product of a linear and a quadratic factor. [2]

(b) Hence solve $x^3+27=(x+3)(x+10)$, expressing non-integer roots in surd form. [3]

(c) Find the value of k given that x^3+27k^3 leaves a remainder of 351 when divided by $x-2$. [2]

3 The diagram shows a *L*-shaped rod *ABC* where *AB* and *BC* have length 18 m and 8 m respectively and angle *ABC* is 90°. The rod is hinged to a wall at *A* so as to rotate in a vertical plane. The rod *AB* makes an acute angle θ with the vertical wall surface *OA*.



(a) Given that *G* is a point directly below *C*, show that $OG = p\cos\theta + q\sin\theta$, where *p* and *q* are constants to be found. [2]

(b) Express *OG* in the form $R\cos(\theta - \alpha)$ where $R > 0$ and $0° \le \alpha \le 90°$. [3]

(c) Find the length of *OG* and the corresponding value of θ if *G* is at maximum displacement from *O*. [3]

4 A circle C_1 has equation $x^2 + y^2 - 6x + 4y = 12$.

(a) Find the radius and the coordinates of the centre of C_1. [3]

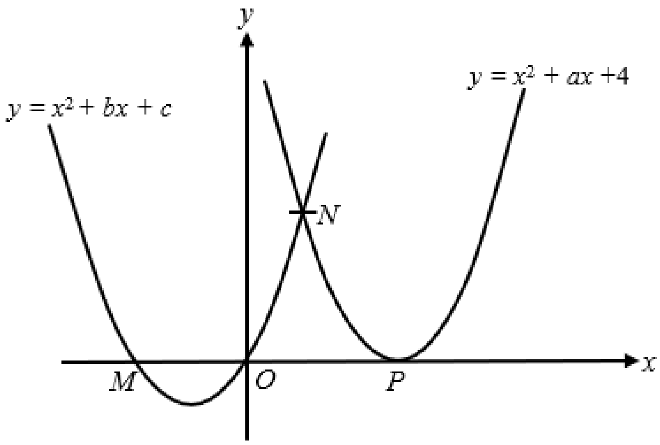
(b) Find the equation of the tangent to the circle at the point $P(7, -5)$. [3]

(c) Another circle C_2 has centre $(-8, 4)$ and radius 7 cm. Find the shortest distance between the 2 circles. [2]

5 **(a)** Prove the identity $\dfrac{(\sin A - \cos A)(1 + \sin A \cos A)}{\cos^3 A} = \tan^3 A - 1$. [4]

(b) Hence solve $(\sin A - \cos A)(1 + \sin A \cos A) + 2\cos^3 A = 0$ exactly, for $-\pi \le A \le \pi$ radians. [4]

6 The diagram shows the graph of $y = x^2 + ax + 4$ and $y = x^2 + bx + c$. The graph of $y = x^2 + ax + 4$ touches the x-axis at P. Points M and O are the x-intercepts of the graph of $y = x^2 + bx + c$. The origin O is the mid-point of MP.



(a) Find the values of a, b and c. [4]

(b) The graph of $y = x^2 + ax + 4$ and $y = x^2 + bx + c$ intersects at *N*. Find the coordinates of *N*. [2]

(c) The graph $y = px^2 + qx + r$ has its turning point at *N* and passes through point *P*. Find the values of *p*, *q* and *r*, where $r > 0$. [3]

7 A particle P, travels in a straight line, so that its displacement, s m, from O at time t seconds, is modelled by $s = \frac{1}{3}t^3 - 5t^2 - 3$.

(a) Find the value of t when particle P returns to its initial position. [2]

(b) Find the minimum velocity of particle P. [3]

(c) Another particle, Q, travels in a straight line from O such that its velocity, v m/s, at time t seconds after passing O is given by $v = 24\left(e^{-\frac{t}{6}} - e^{-1}\right)$.

Find the value of t at which the particle Q is instantaneously at rest. [2]

(d) Find the total distance travelled by particle Q for the first 9 seconds. [4]

8 The height, h cm, of a plant is modelled by $h = \dfrac{80}{1+10e^{-0.4t}}$, where t is the number of months after the first observation.

(a) Show that h is an increasing function. [3]

(b) Find the value of t when the height of the plant first exceeds four times its initial observation. [3]

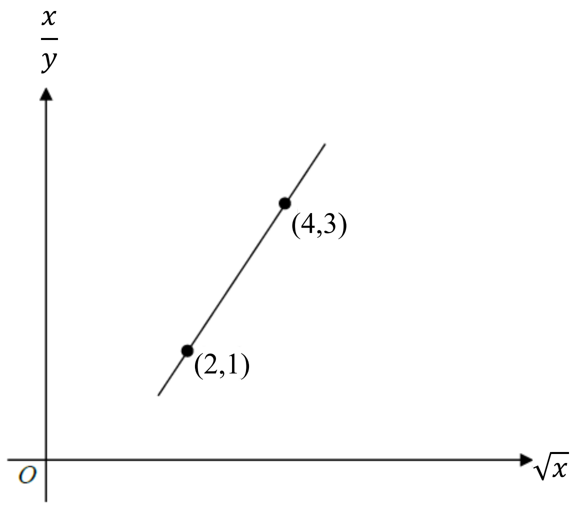
(c) The height, y cm, of another species of plant, t months after the first observation is given by $y = \dfrac{1}{a + b\mathrm{e}^{-t}}$, where a and b are constants. Explain clearly how a straight line graph can be drawn to represent this relationship. You should state which variables should be plotted on each axis and explain how the values of a and b can be calculated. [4]

9 **(a)** Without using a calculator, solve the equation $x\sqrt{15}+\sqrt{5}=x\sqrt{2}+\sqrt{6}$. Leave your answer in the form $p\sqrt{10}+q\sqrt{3}$, where p and q are fractions. [4]

(b) Without using a calculator, solve the equation $\log_2 x-\log_x 16=0$. [4]

(c) Solve the equation $3^{x+2} - 2\left(3^{-x}\right) = 17$. [4]

10 **(a)** The diagram shows part of a straight line graph which passes through $(2,1)$ and $(4,3)$.



Find the equation of the straight line in the form $y = \dfrac{x}{a + b\sqrt{x}}$, where a and b are constants. [3]

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(b) The table below shows the experimental values of two variables x and y.

x	1	2	3	4	5	6
y	63	127	258	510	1000	2100

It is known that x and y are related by an equation of the form $y = \frac{b^x}{10^a}$, where a and b are constants.

(i) On the grid next page, plot $\lg y$ against x and draw a straight line graph. [3]

(ii) Use your graph to estimate the value of a and of b. [3]

(iii) Explain how would you use the graph to find the value of x for which $(10b)^x = 10^{a+1}$. [2]

END OF PAPER

1 It is given that P, Q and R are the angles of a triangle.

(a) Show that $\cos P = -\cos(Q+R)$. [2]

$\cos P$	$= \cos[180° - (Q+R)]$	[M1 – for replacing]
	$= \cos 180° \cos(Q+R) + \sin 180° \sin(Q+R)$	
	$= -\cos(Q+R) + 0$	[A1 – apply addition formula &
	$= -\cos(Q+R)$	[evaluate to arrive at result]

Accept supplementary angles:

$\cos P$	**$= -\cos(180° - P)$**	**M1**
	$= -\cos(Q+R)$	**A1**

(b) Given that $Q = 45°$ and $R = 60°$, find $\cos P$ in the form $\frac{1}{4}(\sqrt{a} - \sqrt{b})$, where a and b are integers. [3]

$\cos P$	$= -\cos(45° + 60°)$	
	$= -\cos 45° \cos 60° + \sin 45° \sin 60°$	[M1 – correct use of formula expansion]
	$= -\frac{\sqrt{2}}{2}(\frac{1}{2}) + \frac{\sqrt{2}}{2}(\frac{\sqrt{3}}{2})$	[M1 – correct special angles trigo ratios]
	$= \frac{1}{4}(\sqrt{6} - \sqrt{2})$	[A1]

2 Baking powder is poured onto a flat surface at a constant rate of 2π cm^3s^{-1} and formed a right circular cone. The radius of the cone is always $\frac{1}{18}$ of its height. Find the rate of change of the radius of the cone after 3 seconds of pouring. [5]

Vol. of cone, $V = \frac{1}{3}\pi r^2(18r)$

$= 6\pi r^3$

$\frac{dV}{dr} = 18\pi r^2$ [B1]

After 3 seconds, $V = 6\pi$

$6\pi r^3 = 6\pi$ [M1 - finding corresponding r]

$r = 1$ [A1]

$\frac{dV}{dt} = \left.\frac{dV}{dr}\right|_{r=1} \times \left.\frac{dr}{dt}\right|_{r=1}$ [M1 - connected rate of change]

$2\pi = 18\pi(1)^2 \times \left.\frac{dr}{dt}\right|_{r=1}$

$\left.\frac{dr}{dt}\right|_{r=1} = \frac{1}{9}$

The rate of change required is $= \frac{1}{9}$ cm/s. [A1 o.e.]

3 **(a)** Determine the set of values of m for which the equation $2x^2+4x+2m=6mx-2$ has real roots. [4]

$$2x^2+(4-6m)x+2m+2=0$$

$$b^2-4ac\geq 0$$ [M1 - correct Discriminant]

$$(4-6m)^2-4(2)(2m+2)\geq 0$$

$$16-48m+36m^2-8(2m+2)\geq 0$$

$$36m^2-64m\geq 0$$ [M1 – simplification in factors]

$$m(9m-16)\geq 0$$

$$m\leq 0 \quad or \quad m\geq\frac{16}{9}$$ [A2, minus 1 mark if inequality sign is wrong due to earlier wrong D sign]

(b) Hence state what can be deduced about the curve $y=2(x+1)^2$ and the line $y=6x-2$. Justify your statement. [2]

$2x^2+4x+2=6x-2$

By comparing with (a), m = 1 [B1 – correct m value]

When m = 1, it is not within the set of values of m for which there will be real roots, hence the curve will not meet the line/ the curve will not cut the line. [B1]

4 **(a)** Show that $\frac{d}{dx}(\ln(\cos x)) = -\tan x$. [2]

$\frac{d}{dx}(\ln(\cos x)) = \frac{1}{\cos x} \times \frac{d}{dx}(\cos x)$	[M1 – show working]
$= \frac{-\sin x}{\cos x} = -\tan x$	[A1 – show fraction]

(b) Differentiate $x \tan x$ with respect to x. [2]

$\frac{d}{dx} x\tan x = x\sec^2 x + \tan x$	[M1 – show product rule] [A1 – correct ans for both]

(c) Using the results from part **(a)** and **(b)**, find $\int x\sec^2 x \, dx$ and hence show that $\int_0^{\frac{\pi}{4}} x\sec^2 x \, dx = \frac{\pi}{4} - \frac{1}{2}\ln 2$. [4]

From (b), $\int (x\sec^2 x + \tan x) \, dx = x\tan x + C$ $\int x\sec^2 x \, dx + \int \tan x \, dx = x\tan x + C$	[M1 – use part (b), 'C' must be seen]
$\int x\sec^2 x \, dx = x\tan x - \int \tan x \, dx + C$ $= x\tan x + \ln(\cos x) + C$	[M1 – proper integration and final correct ans, 'C' must be seen]
$\int_0^{\frac{\pi}{4}} x\sec^2 x \, dx = \left[x\tan x + \ln(\cos x)\right]_0^{\frac{\pi}{4}}$	[minus 1 mark if "C" is not seen]
$= \frac{\pi}{4}\tan\frac{\pi}{4} + \ln\left(\cos\frac{\pi}{4}\right) - 0 - \ln 1$ $= \frac{\pi}{4} + \ln\left(\frac{1}{\sqrt{2}}\right)$	[M1 – proper evaluation]
$= \frac{\pi}{4} + \ln 2^{-\frac{1}{2}}$ $= \frac{\pi}{4} - \frac{1}{2}\ln 2$	[A1]

5 **(a)** In the expansion of $(2+x)^n$, where n is a positive integer, the coefficient of x^2 is twice the coefficient of x. Find the value of n. [3]

$$(2+x)^n = 2^n + \binom{n}{1}2^{n-1}x + \binom{n}{2}2^{n-2}x^2 + \ldots$$

$\binom{n}{2}2^{n-2} = 2\binom{n}{1}2^{n-1}$ [M1 – correct coeff]

$\frac{n(n-1)}{2}\left(2^{n-2}\right) = 2n\left(2^{n-1}\right)$

$\frac{n(n-1)}{2}\left(2^n \times 2^{-2}\right) = 2n\left(2^n \times 2^{-1}\right) \quad (n \neq 0)$ [M1 – simplify]

$\frac{n-1}{2(4)} = 1$

$n-1=8$ [A1 – with rejection]

$n=9$

(b) Find the value of the term that is independent of x in the expansion of $\left(2x - \frac{1}{4x^4}\right)^{15}$. [4]

general term

$= \binom{15}{r}(2x)^{15-r}\left(-\frac{1}{4x^4}\right)^r$

$= \binom{15}{r}2^{15-r}x^{15-r}\left(-\frac{1}{4}\right)^r x^{-4r}$ [M1]

$= \binom{15}{r}2^{15-r}\left(-\frac{1}{4}\right)^r x^{15-5r}$ [M1 - gather x and let power = 0]

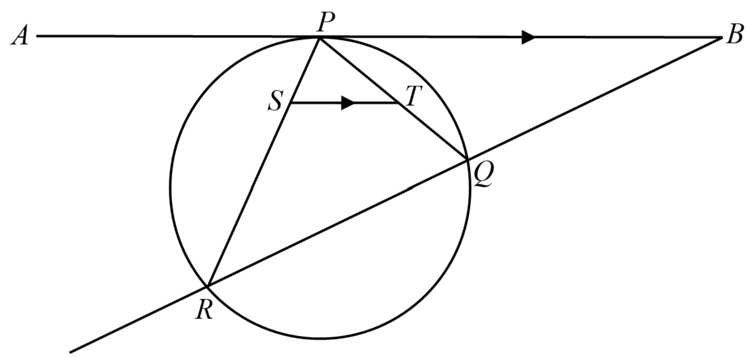
$15-5r=0$

$r=3$ [M1 for r value]

value $= \binom{15}{3}2^{12}\left(-\frac{1}{4}\right)^3 = -29120$ [A1]

Max 2 marks if able to gather 15 – 5r for exponent of 'x' and equate to zero with correct r value.

6



The diagram shows a circle passing through the points P, Q and R. The point Q lies on the line RB. AB is a tangent to the circle at P. The points S and T lie on PR and PQ respectively. Given that AB is parallel to ST, prove that

(a) triangle PST is similar to triangle PQR, [3]

$\angle SPT = \angle QPR$ (common angle) [M1]

$\angle PST = \angle SPA$ (alt. $\angle$s, AB // ST) **OR** $\angle PTS = \angle TPB$ (alt. $\angle$s, AB // ST)

$= \angle PQR$ (Alt. Segment Thm) $= \angle PRQ$ (Alt. Segment Thm) [M1]

$\therefore$ triangle PST is similar to triangle PQR. [A1]

(2 pairs of corresponding angles are equal)

(b) $PQ \times PT = PR \times PS$, [2]

From above result, $\dfrac{PQ}{PS} = \dfrac{PR}{PT}$ [M1]

$\Rightarrow PQ \times PT = PR \times PS$ [A1]

(c) Determine if $STQR$ is a cyclic quadrilateral. [4]

$QTS = 180° - \angle PTS$ (adj. $\angle$s on a st. line)

$= 180° - \angle QRS$ (from (a) result) [M1]

So $\angle QTS + \angle QRS = 180°$ [M1]

$\angle RST + \angle RQT = 360° - (\angle QTS + \angle QRS)$ ($\angle$ sum of a quadrilateral)

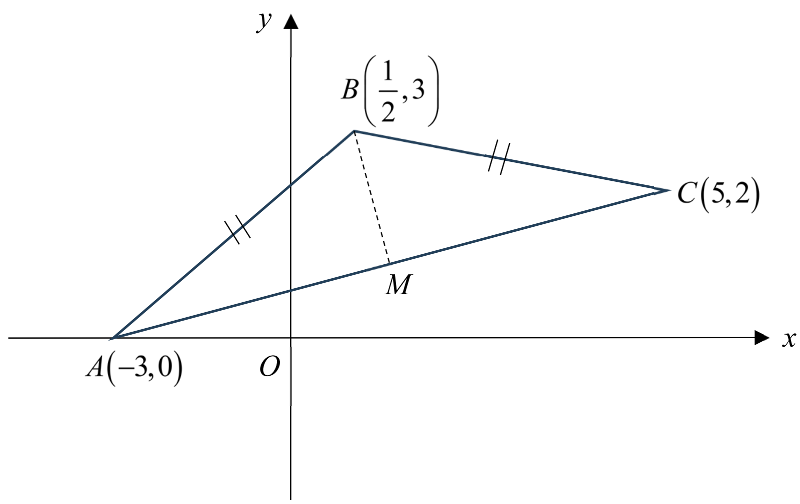
$= 360° - 180°$

$= 180°$ [M1]

By converse of angles in opposite segment, $STQR$ is a cyclic quadrilateral and all four vertices lie on the circumference of a circle.

[A1 – with correct reason, accept even if no mention of four vertices]

7



The diagram shows an isosceles triangle ABC in which $A(-3,0)$, $B\left(\frac{1}{2},3\right)$ and $C(5,2)$. M is the foot of perpendicular from B to AC.

(a) Find the coordinates of M. [1]

$M = \left(\frac{-3+5}{2}, \frac{0+2}{2}\right) = (1, 1)$ [B1]

(b) Find the equation of the perpendicular bisector of AC. [2]

Gradient of $AC = \frac{2-0}{5-(-3)} = \frac{1}{4}$

Gradient of perpendicular BM = – 4 [M1 – use $m \times m_{\perp} = -1$]

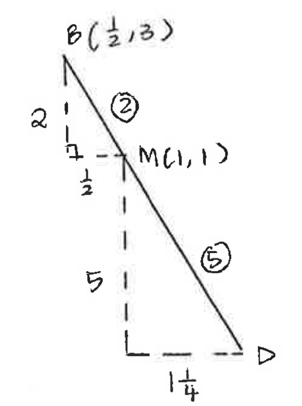
or $m_{BM} = \frac{3-1}{\frac{1}{2}-1} = -4$

Equation of perpendicular bisector:

$y - 1 = -4(x-1)$

$y = -4x + 5$ [A1]

(c) Given that *ABCD* is a kite with $BM = \frac{2}{7}BD$, find the coordinates of *D*. [3]



Correct ratio: 2 : 5

$\frac{1}{2} : 1\frac{1}{4}$ M1

Coordinates of D

$= \left(1+\frac{1}{4}, 1-5\right)$ M1

$= \left(2\frac{1}{4}, -4\right)$ A1

Alternative: using vectors

$\overrightarrow{BM} = \frac{2}{7}\overrightarrow{BD}$

$\begin{pmatrix} 0.5 \\ -2 \end{pmatrix} = \frac{2}{7}\left(\overrightarrow{OD} - \overrightarrow{OB}\right)$ [M1 - position vectors]

$\overrightarrow{OD} = 3.5\begin{pmatrix} 0.5 \\ -2 \end{pmatrix} + \begin{pmatrix} 0.5 \\ 3 \end{pmatrix} = \begin{pmatrix} 2.25 \\ -4 \end{pmatrix}$ [M1]

D= $\left(2\frac{1}{4}, -4\right)$ [A1]

(d) Find the area of the kite *ABCD*. [2]

Area of *ABCD* = $\frac{1}{2}\begin{vmatrix} -3 & 2\frac{1}{4} & 5 & \frac{1}{2} & -3 \\ 0 & -4 & 2 & 3 & 0 \end{vmatrix}$

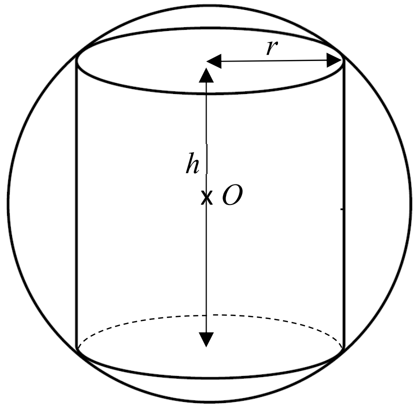
M1 (ecf 1 – correct method)

$= \frac{1}{2}(31.5 + 28)$ (anti-clockwise)

$= 29.75$ units^2 (or $\frac{119}{4}$) A1

(accept 29.7 with 3 s.f. if using other method such as Pythagoras'

8



A prototype consists of a cylindrical container of height h cm and radius r cm inscribed in a hollow sphere with centre O.

The sphere has a surface area of 6400π cm^2 and both the sphere and container have negligible thickness.

(a) Show that the volume of the cylinder container, V cm^3, is given by [3]

$$V = 2\pi r^2\sqrt{1600 - r^2}\ .$$

$4\pi R^2 = 6400\pi$

$R = 40$ (radius R of sphere) [M1 – find radius R]

By Pythagoras' Theorem,

$r^2 + \left(\frac{h}{2}\right)^2 = 40^2$ [M1]

$h^2 = 4\left(1600 - r^2\right)$

$h = \sqrt{4\left(1600 - r^2\right)}$

$h = 2\sqrt{1600 - r^2}$

$V = \pi r^2 h$

$= 2\pi r^2\sqrt{1600 - r^2}$ [A1]

(b) Given that r can vary, find the value of r for which the volume V is stationary. [5]

$$\frac{dV}{dr} = (2\pi r^2)\times\frac{1}{2}\left(1600-r^2\right)^{-\frac{1}{2}}\times(-2r)+\sqrt{1600-r^2}\times 2\times 2\pi r$$ [M2 – correct differentiation using product rule, deduct M1 if either term is incorrect]

$$\frac{dV}{dr} = 4\pi r\sqrt{1600-r^2}-\frac{2\pi r^3}{\sqrt{1600-r^2}}$$

For stationary values, $\frac{dV}{dr}=0$

$$4\pi r\sqrt{1600-r^2}-\frac{2\pi r^3}{\sqrt{1600-r^2}}=0$$ [M1 – equate to zero]

$$\frac{2\pi r}{\sqrt{1600-r^2}}\left[2\left(1600-r^2\right)-r^2\right]=0$$ [M1 – simplify and show this or the next line working]

$$\frac{2\pi r}{\sqrt{1600-r^2}}\left[3200-3r^2\right]=0$$

$r=0$ (reject) or $3200-r^2=0$

$r=\frac{40\sqrt{6}}{3}=32.7$ cm [A1]

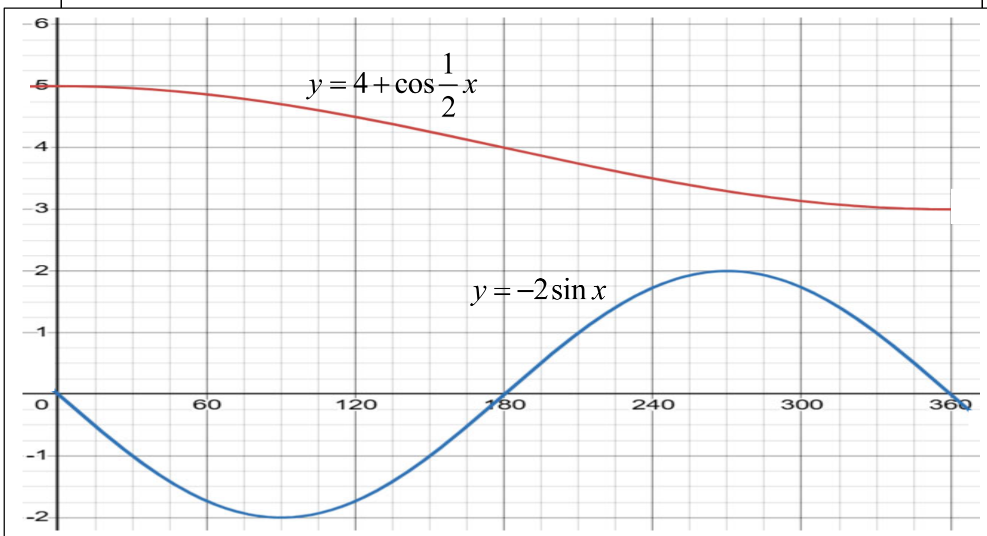
(c) A scientist plans to launch this prototype into outer space carrying as much fuel as possible. Explain whether the prototype can satisfy the scientist's requirement. [2]

x	$\left(\frac{40\sqrt{6}}{3}\right)^-$	$\frac{40\sqrt{6}}{3}$	$\left(\frac{40\sqrt{6}}{3}\right)^+$
$\frac{dV}{dx}$	+	0	–
sketch of tangent	/	—	\

[M1 – first or 2nd derivative, ecf 1 for method]

Since this value of r give maximum V value, the prototype satisfies the scientist's requirement as he could maximise the fuel to be stored in the prototype. [B1]

9	It is given that $f(x) = 4 + \cos\left(\frac{x}{2}\right)$ and $g(x) = -2\sin x$.		
	(a)	State the period and amplitude of $f(x)$.	[2]
		Period = 720° or 4π [B1] Amplitude = 1 [B1]	
	(b)	State the period and amplitude of $g(x)$.	[1]
		Period = 360° or 2π Amplitude = 2 [B1 for both]	
	(c)	Sketch, on the same axes, the graphs of $y = f(x)$ and $y = g(x)$ for $0° \le x \le 360°$.	[3]



	f(x)	g(x)
Shape	cosine	negative sine
Amplitude	1	2
No. of cycle	Half	1
Shift	4	-

[B1 – for starts and ends at 'zero' for $g(x)$]

[B1 – for starts '5' and ends at '3' for $f(x)$]

[B1 – fully correct graphs]

10 **(a)** Express $\dfrac{1-3x-3x^2}{x(x+1)^2}$ in partial fractions. [5]

$$\frac{1-3x-3x^2}{x(x+1)^2}=\frac{A}{x}+\frac{B}{x+1}+\frac{C}{(x+1)^2}$$

$1-3x-3x^2=A(x+1)^2+Bx(x+1)+Cx$ [M1]

When $x = -1$, $C=-1$ [M1]

When $x=0$, $A=1$ [M1]

When $x=1$, $B=-4$ [M1]

$$\frac{1-3x-3x^2}{x(x+1)^2}=\frac{1}{x}-\frac{4}{x+1}-\frac{1}{(x+1)^2}$$ [A1]

(b) Hence find $\displaystyle\int\frac{1-3x-3x^2}{2x(x+1)^2}\,dx$. [4]

$$\int\frac{1-3x-3x^2}{2x(x+1)^2}\,dx=\frac{1}{2}\int\frac{1-3x-3x^2}{x(x+1)^2}\,dx$$

$$=\frac{1}{2}\int\frac{1}{x}-\frac{4}{x+1}-\frac{1}{(x+1)^2}\,dx$$ [M1 – take out 0.5]

$$=\frac{1}{2}\ln x-2\ln(x+1)+\frac{1}{2(x+1)}+C$$ [A1for each term, must have constant C]

Accept $=\ln\sqrt{x}-\ln(x+1)^2+\dfrac{1}{2(x+1)}+C$

or $\ln\dfrac{\sqrt{x}}{(x+1)^2}+\dfrac{1}{2(x+1)}+C$

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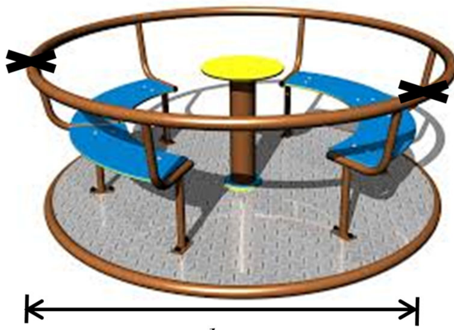


The horizontal distance of a child on a carousel, h m, from the starting point is modelled by the equation, $h = 2(1 - \cos kt)$, where k is a constant and t is the time in seconds after the child leaves the starting point. The time to complete one revolution is 20 seconds.

(a) Explain why this model suggests that the diameter of the carousel is 4 m. [1]

$h = 2(1 - \cos kt)$.

Since the diameter of the carousel = max value of h, $h = 2\,(1 - (-1)) = 4$ m

[B1 – must relate diameter to *h, do not accept amplitude method as question ask on the model equation*]

(b) Show that the value of k is $\frac{\pi}{10}$ radians per second. [2]

Period = 20 s

$\frac{2\pi}{k} = 20$ [M1]

$k = \frac{\pi}{10}\, rad/s$ (*shown*) [A1]

accept
2π rad in 20 s
$\frac{2\pi}{20}$ rad in 1 s
$\frac{2\pi}{20}t$ rad in t s
$k = \frac{\pi}{10}$

accept
$h = 4$ when $t = 10$
to find k value

(c) As the carousel turns, it is possible for the child on the carousel to view a landmark, provided that the horizontal distance of the child is within 1 m from the starting point.

Find the duration of time for which the child will not be able to view the landmark during one revolution. [5]

$1 = 2(1-\cos\frac{\pi}{10}t)$

accept $2(1-\cos\frac{\pi}{10}t) > 1$

[M1]

$\frac{1}{2} = \cos\frac{\pi}{10}t$

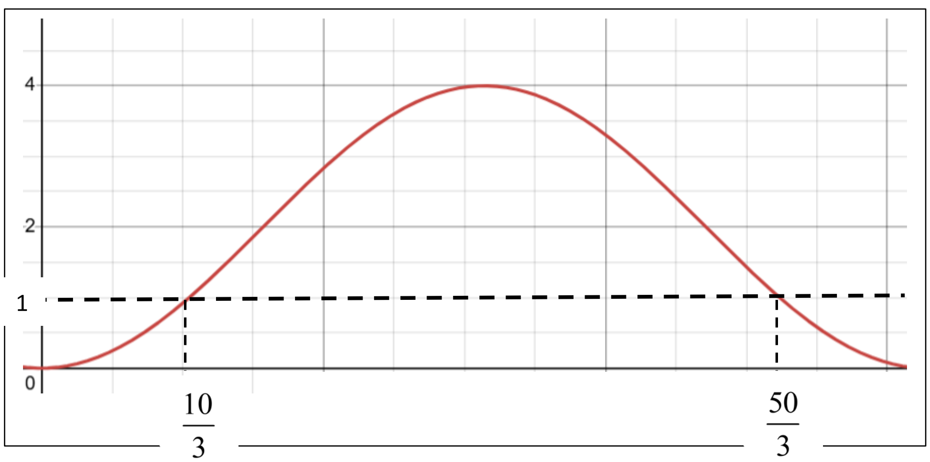
$basic\angle = \frac{\pi}{3}$ (solution in 1st and 4th quad)

$\frac{\pi}{10}t = \frac{\pi}{3}, \frac{5\pi}{3}$ [M1]

$t = \frac{10}{3}, \frac{50}{3}$ [M1]

accept $t > \frac{10}{3}$ $t < \frac{50}{3}$

Not able to view: $\frac{50}{3} - \frac{10}{3} = \frac{40}{3}s$ *or* $13.3\ s\ (3s.f.)$ [M1, A1]



[Turn over

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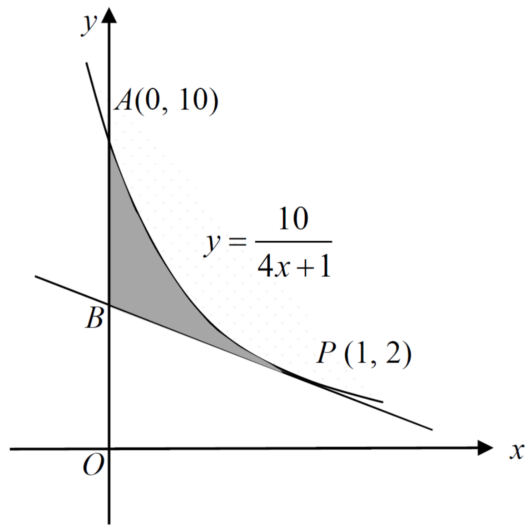


The diagram shows part of the curve $y=\frac{10}{4x+1}$ intersecting the y-axis at $A(0, 10)$. The tangent to the curve at the point $P(1, 2)$ intersects the y-axis at B.

(a) Show that the coordinates of B is (0, 3.6). [4]

$y=\frac{10}{4x+1}=10(4x+1)^{-1}$

$\frac{dy}{dx}=-10(4x+1)^{-2}(4)$ [M1]

$\frac{dy}{dx}=-40(4x+1)^{-2}$

When $x=1$ $\frac{dy}{dx}=-40(4(1)+1)^{-2}$

$\frac{dy}{dx}=-1.6$ [M1]

$\frac{y-2}{0-1}=-1.6$ [M1]

$y-2=1.6$

$y=3.6$

Coordinate of B is (0, 3.6) [A1]

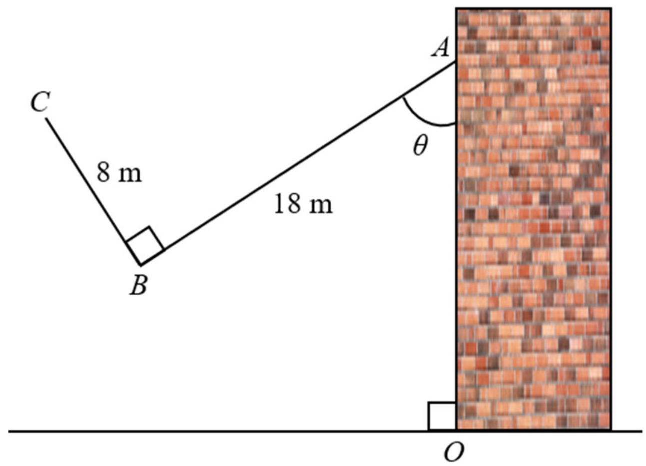
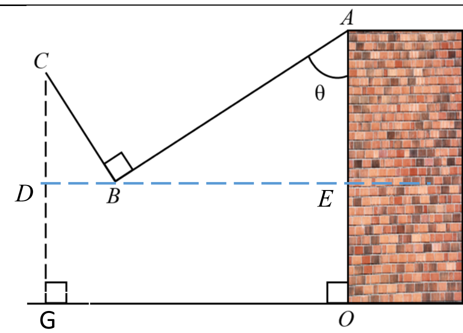
(b) Find the **exact** area of the shaded region. [5]

$Area = \int_0^1 \frac{10}{4x+1}\, dx - \frac{1}{2}(3.6+2)(1)$	[M1], [M1]
$Area = \left[\frac{10\ln(4x+1)}{4}\right]_0^1 - 2.8$	[M1]
$Area = \left[\frac{10\ln(4+1)}{4} - \frac{10\ln(1)}{4}\right] - 2.8$	[M1]
$Area = \frac{5}{2}\ln 5 - 2.8 \text{ unit}^2$	[A1]

END OF PAPER

1	A curve has the equation $y=\dfrac{\sin 2x}{2-\cos 2x}$.
	(a) Show that the gradient function can be expressed in the form $\dfrac{k\cos 2x-2}{(2-\cos 2x)^2}$, where k is a constant. [3]
	$y=\dfrac{\sin 2x}{2-\cos 2x}$ $\dfrac{dy}{dx}=\dfrac{(2-\cos 2x)2\cos 2x-\sin 2x(2\sin 2x)}{(2-\cos 2x)^2}$ — M1: correct quotient/pdt rule $=\dfrac{4\cos 2x-2\cos^2 2x-2\sin^2 2x}{(2-\cos 2x)^2}$ — M1: differentiate sin 2x and cos 2x correctly $=\dfrac{4\cos 2x-2(\cos^2 2x+\sin^2 2x)}{(2-\cos 2x)^2}$ — A1: use of identity to reach answer $=\dfrac{4\cos 2x-2}{(2-\cos 2x)^2}$
	(b) Find the acute angle between the tangent to the curve at $x=\dfrac{\pi}{12}$ and the line $y=0$. [3]
	gradient of tangent $=\dfrac{4\cos\frac{\pi}{6}-2}{\left(2-\cos\frac{\pi}{6}\right)^2}=1.1386$ — M1: correct gradient value Angle required $=\tan^{-1}(1.1386)=48.7°$ (1 dp) — M1, A1 (accepts 0.850 rad)

2	**(a)**	Factorise x^3+27k^3 as a product of a linear and a quadratic factor. [2]
		$x^3+27k^3=(x+3k)(x^2-3kx+9k^2)$ M1: $(x+3k)$ A1
	(b)	Hence solve $x^3+27=(x+3)(x+10)$, expressing non-integer roots in surd form. [3]
		$k=1: x^3+27=(x+3)(x^2-3x+9)$ $(x+3)(x^2-3x+9)=(x+3)(x+10)$ $x+3=0$ or $x^2-3x+9=x+10$ $x=-3$ or $x^2-4x-1=0$ $x=\frac{4\pm\sqrt{16-4(-1)}}{2}$ $=2\pm\sqrt{5}$ B1: identify x=-3 as a root from Hence M1: apply quad formula correctly A1
	(c)	Find the value of k given that x^3+27k^3 leaves a remainder of 351 when divided by $x-2$. [2]
		Let $f(x)=x^3+27k^3$ $f(2)=2^3+27k^3=351$ $k=\sqrt[3]{\frac{351-8}{27}}=\frac{7}{3}$ M1: applying remainder thm correctly A1

3	The diagram shows a *L*-shaped rod *ABC* where *AB* and *BC* have length 18 m and 8 m respectively and angle *ABC* is 90°. The rod is hinged to a wall at *A* so as to rotate in a vertical plane. The rod *AB* makes an acute angle θ with the vertical wall surface *OA*. 
	(a) Given that *G* is a point directly below *C*, show that $OG = p\cos\theta + q\sin\theta$, where *p* and *q* are constants to be found. [2]
	

Using triangle *BCD*: Angle $DBC = \theta$

$$\cos\theta = \frac{BD}{8}$$

$$BD = 8\cos\theta$$

B1: either correctly establishing BD or BE

Using triangle *ABE*: Angle $BAE = \theta$

$$\sin\theta = \frac{BE}{18}$$

$$BE = 18\sin\theta$$

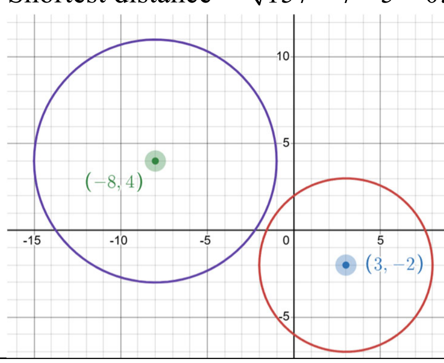
$OG = BD + BE$

$= 8\cos\theta + 18\sin\theta$, $p = 8$, $q = 18$

B1: showing the other component and clear indication that OG is a sum of the 2 values

(b) Express *OG* in the form $R\cos(\theta - \alpha)$ where $R > 0$ and $0° \leq \alpha \leq 90°$. [3]

	$OG = 8\cos\theta + 18\sin\theta$ $= \sqrt{8^2+18^2}\cos\left(\theta - \tan^{-1}\frac{18}{8}\right)$ $= \sqrt{388}\cos\left(\theta - \tan^{-1}\frac{9}{4}\right)$ $= 19.7\cos(\theta - 66.0°)$	M1: $\sqrt{8^2+18^2}$ M1: $\tan^{-1}\frac{18}{8}$ A1: correct evaluation and form, accept $\sqrt{388}$
	(c) Find the length of *OG* and the corresponding value of θ if *G* is at maximum displacement from *O*. [3]	
	max $OG = \sqrt{388}$ $= 19.7$ m when $\cos\left(\theta - \tan^{-1}\frac{9}{4}\right) = 1$ $\theta - \tan^{-1}\frac{9}{4} = 0$ $\theta = \tan^{-1}\frac{9}{4} = 66.0°$	A1: $\sqrt{388}$ or $19.7m$ M1: $\theta - \tan^{-1}\frac{9}{4} = 0$ A1: 66.0°

4	A circle C_1 has equation $x^2+y^2-6x+4y=12$.		
	(a) Find the radius and the coordinates of the centre of C_1. [3]		
	$x^2-6x+y^2+4y=12$ $(x-3)^2+(y+2)^2=12+3^3+2^2$ $(x-3)^2+(y+2)^2=5^2$	M1: completing the square or using formula	
	Radius = 5 units Centre = (3, -2)	A1 B1	
	(b) Find the equation of the tangent to the circle at the point $P(7,-5)$. [3]		
	Gradient of normal = $\frac{-2-(-5)}{3-7}=-\frac{3}{4}$	M1	
	Gradient of tangent = $\frac{4}{3}$		
	Eqn of tangent: $y+5=\frac{4}{3}(x-7)$	M1award for correct pts and their -1/m used	
	$y=\frac{4}{3}x-\frac{43}{3}$	A1, or $3y = 4x - 43$	
	(c) Another circle C_2 has centre $(-8, 4)$ and radius 7 cm. Find the shortest distance between the 2 circles. [2]		
	Distance between centres of circle = $\sqrt{(3+8)^2+(-2-4)^2}=\sqrt{157}$ Shortest distance = $\sqrt{157}-7-5=0.530$ cm (3 s.f.)	M1 A1	
			

[Turn over

5 **(a)** Prove the identity $\frac{(\sin A - \cos A)(1+\sin A\cos A)}{\cos^3 A} = \tan^3 A - 1$. [4]

$$LHS = \frac{(\sin A - \cos A)(1+\sin A\cos A)}{\cos^3 A}$$

$$= \frac{\sin A - \cos A + \sin^2 A\cos A - \sin A\cos^2 A}{\cos^3 A}$$

M1: correct expansion of terms

$$= \frac{\sin A - \cos A + (1-\cos^2 A)\cos A - \sin A(1-\sin^2 A)}{\cos^3 A}$$

M1: applying identity

$$= \frac{\sin A - \cos A + \cos A - \cos^3 A - \sin A + \sin^3 A}{\cos^3 A}$$

$$= \frac{\sin^3 A - \cos^3 A}{\cos^3 A}$$

M1: expand & simplify

$$= \frac{\sin^3 A}{\cos^3 A} - \frac{\cos^3 A}{\cos^3 A}$$

A1: manipulation to RHS

$$= \tan^3 A - 1$$

OR

$$LHS = \frac{(\sin A - \cos A)(1+\sin A\cos A)}{\cos^3 A}$$

$$= \frac{(\sin A - \cos A)(\sin^2 A + \cos^2 A + \sin A\cos A)}{\cos^3 A}$$

M1: applying identity

$$= \frac{\sin^3 A + \sin A\cos^2 A + \sin^2 A\cos A - \sin^2 A\cos A - \cos^3 A - \sin A\cos^2 A}{\cos^3 A}$$

M1: correct expansion of terms

$$= \frac{\sin^3 A - \cos^3 A}{\cos^3 A}$$

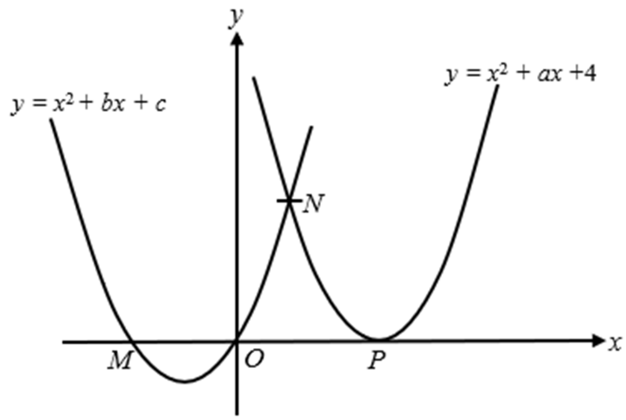
M1: simplify

$$= \frac{\sin^3 A}{\cos^3 A} - \frac{\cos^3 A}{\cos^3 A}$$

A1: manipulation to RHS

$$= \tan^3 A - 1$$

	(b) Hence solve $(\sin A - \cos A)(1 + \sin A \cos A) + 2\cos^3 A = 0$ exactly, for $-\pi \le A \le \pi$ radians. [4]
	$(\sin A - \cos A)(1 + \sin A \cos A) = -2\cos^3 A$ $\dfrac{(\sin A - \cos A)(1 + \sin A \cos A)}{\cos^3 A} = -2$ $\tan^3 A - 1 = -2$ — M1: simplification to single trigo equation $\tan^3 A = -1$ $\tan A = -1$ Basic angle = $\dfrac{\pi}{4}$ — M1: correct basic angle – must be acute Quad: Q2, Q4 $A = \dfrac{3\pi}{4}, -\dfrac{\pi}{4}$ — A1, A1

6	The diagram shows the graph of $y = x^2 + ax + 4$ and $y = x^2 + bx + c$. The graph of $y = x^2 + ax + 4$ touches the x-axis at P. Points M and O are the x-intercepts of the graph of $y = x^2 + bx + c$. The origin O is the mid-point of MP. 
	(a) Find the values of a, b and c. [4]
	Since origin (0, 0) is on $y = x^2 + bx + c$, $c = 0$ — B1 Discriminant of $y = x^2 + ax + 4 = 0$ since curve intersects x-axis once only: $a^2 - 4(1)(4) = 0$ $a = \pm 4$ Since P is on positive x-axis, $a < 0$: $a = -4$ M1: using discriminant or the equation must be a perfect square A1 OR $y = x^2 + ax + 4 = \left(x + \frac{a}{2}\right)^2 - \frac{a^2}{4} + 4$. Since curve intersects x-axis once only: $4 - \frac{a^2}{4} = 0$ $a = \pm 4$ Since P is on positive x-axis, $a < 0$: $a = -4$ M1: using discriminant or the equation must be a perfect square A1 Coor of P = $\left(-\frac{a}{2}, 0\right) = (2, 0)$ Coor of M = $(-b, 0) = (-2, 0)$ $b = 2$ — B1

	(b) The graph of $y = x^2 + ax + 4$ and $y = x^2 + bx + c$ intersects at N. Find the coordinates of N. [2]
	$x^2 - 4x + 4 = x^2 + 2x$ $6x = 4$ $x = \frac{2}{3}$ M1: equating and solving correct x (allows ecf) $y = \left(\frac{2}{3}\right)^2 + 2\left(\frac{2}{3}\right) = \frac{16}{9}$ Coordinates of $N = \left(\frac{2}{3}, \frac{16}{9}\right)$ A1
	(c) The graph $y = px^2 + qx + r$ has its turning point at N and passes through point P. Find the values of p, q and r, where $r > 0$. [3]
	Graph with turning point at N and passes P (downward opening): $y = -p\left(x - \frac{2}{3}\right)^2 + \frac{16}{9}$, $p < 0$ M1: correct completed square form At P $(2,0)$, $0 = -p\left(2 - \frac{2}{3}\right)^2 + \frac{16}{9}$ $p = -1$ $y = -\left(x - \frac{2}{3}\right)^2 + \frac{16}{9}$ $= -\left(x^2 - \frac{4}{3}x + \frac{4}{9}\right) + \frac{16}{9}$ $= -x^2 + \frac{4}{3}x + \frac{4}{3}$ A1: correct p $p = -1, \; q = r = \frac{4}{3}$ A1: correct q & r OR (longer method): form 3 equations with coor of N, P and either 2nd x-intercept or derivative and solve $y = px^2 + qx + r$ $\frac{dy}{dx} = 2px + q$ Turning point at $\left(\frac{2}{3}, \frac{16}{9}\right)$: $2p\left(\frac{2}{3}\right) + q = 0$ $4p = -3q$ ---- (1) M1: establishing 3 equations correctly Graph passes through P: $0 = p(2)^2 + q(2) + r$ $4p + 2q + r = 0$

	Subst (1): $q = r$ ----- (2) Graph passes through N: $\frac{16}{9} = p\left(\frac{2}{3}\right)^2 + q\left(\frac{2}{3}\right) + r$ $4p + 6q + 9r = 16$ Subst (1) and (2): $-3q + 6q + 9q = 16$ $12q = 16$ $q = \frac{4}{3}$ A1: correct p A1: correct q & r
7	A particle *P*, travels in a straight line, so that its displacement, *s* m, from *O* at time *t* seconds, is modelled by $s = \frac{1}{3}t^3 - 5t^2 - 3$.
	(a) Find the value of *t* when particle *P* return to its initial position. [2]
	Initial position when $t = 0$, $s = -3$ $\frac{1}{3}t^3 - 5t^2 - 3 = -3$ M1: setting s = -3 $\frac{1}{3}t^3 - 5t^2 = 0$ $t^2\left(\frac{1}{3}t - 5\right) = 0$ $t = 0$ or $t = 15s$ Particle returns to initial position at $t = 15$s. A1
	(b) Find the minimum velocity of particle *P*. [3]
	$v = t^2 - 10t$ $= (t-5)^2 - 25$ Minimum velocity = -25 m/s[2] M1: $v = t^2 - 10t$ M1: completing the square A1 OR $v = t^2 - 10t$ At stationary *v*, $\frac{dv}{dt} = 2t - 10 = 0$ $t = 5$ $v = (5)^2 - 10(5) = -25$ m/s M1: $v = t^2 - 10t$ M1: solving for t = 5 $\frac{d^2v}{dt^2} = 2 > 0$, minimum *v* at -25m/s A1: includes check for minimum

(c) Another particle, Q, travels in a straight line from O such that its velocity, v m/s, at time t seconds after passing O is given by $v = 24\left(e^{-\frac{t}{6}} - e^{-1}\right)$.

Find the value of t at which the particle Q is instantaneously at rest. [2]

$v = 24\left(e^{-\frac{t}{6}} - e^{-1}\right) = 0$

$e^{-\frac{t}{6}} = e^{-1}$

$t = 6s$

M1: Setting $v = 0$

A1

(d) Find the total distance travelled by particle Q for the first 9 seconds. [4]

$s = 24\left(\frac{e^{-\frac{t}{6}}}{-\frac{1}{6}} - te^{-1}\right) = -24\left(6e^{-\frac{t}{6}} + te^{-1}\right) + C$

M1: correct integration (award even if without +C)

When t=0, s=0: $s = -24(6) + C = 0$

$C = 144$

$s = 144 - 24\left(6e^{-\frac{t}{6}} + te^{-1}\right)$

M1: correct S

When $t = 6$, $s = 144 - 24\left(6e^{-1} + 6e^{-1}\right) = 38.051m$ (5 sf)

When $t = 9$, $s = 144 - 24\left(6e^{-\frac{9}{6}} + 9e^{-1}\right) = 32.407m$ (5 sf)

M1: sub t= 6 and t=9

Total distance travelled = $s = 38.051 + (38.051 - 32.407) = 43.7m$ (3 sf)

A1

OR

Displacement covered for 1st 6 sec =

$\int_0^6 24\left(e^{-\frac{t}{6}} - e^{-1}\right) dt = 24\left[\frac{e^{-\frac{t}{6}}}{-\frac{1}{6}} - te^{-1}\right]_0^6$

M1: correct integration

$= -24\left[6e^{-\frac{t}{6}} + te^{-1}\right]_0^6$

$= -24\left[6e^{-\frac{6}{6}} + 6e^{-1} - 6e^{-\frac{0}{6}} - 0\right]$

$= 38.051m$ (3sf)

M1

Displacement covered for next 3 sec =

	$\int_6^9 24\left(e^{-\frac{t}{6}}-e^{-1}\right)\mathrm{d}t=-24\left[6e^{-\frac{t}{6}}+te^{-1}\right]_6^9$ $=-24\left[6e^{-\frac{9}{6}}+9e^{-1}-6e^{-\frac{6}{6}}-6e^{-\frac{6}{6}}\right]$ $=-5.6434m$ (5sf) M1 Total distance = $s=38.051+(38.051-32.407)=43.7m$ A1

8	The height, h cm, of a plant is modelled by $h=\dfrac{80}{1+10\mathrm{e}^{-0.4t}}$, where t is the number of months after the first observation.
	(a) Show that h is an increasing function. [3]
	$\dfrac{\mathrm{d}h}{\mathrm{d}t}=80\dfrac{\mathrm{d}}{\mathrm{d}t}\left(1+10\mathrm{e}^{-0.4t}\right)^{-1}$ $=80(-1)\left(1+10\mathrm{e}^{-0.4t}\right)^{-2}\left(10(-0.4)\mathrm{e}^{-0.4t}\right)$ M1: differentiate $1+10\mathrm{e}^{-0.4t}$ correctly $=\dfrac{320\mathrm{e}^{-0.4t}}{\left(1+10\mathrm{e}^{-0.4t}\right)^2}$ M1: simplification Since $\mathrm{e}^{-0.4t}>0$ for $t>0$, $320\mathrm{e}^{-0.4t}>0$ and $\left(1+10\mathrm{e}^{-0.4t}\right)^2>0$ $\dfrac{\mathrm{d}h}{\mathrm{d}t}=\dfrac{320\mathrm{e}^{-0.4t}}{\left(1+10\mathrm{e}^{-0.4t}\right)^2}>0$ Therefore, h is an increasing function. A1: Clear explanation on why derivative is positive and therefore h is increasing
	(b) Find the value of t when the height of the plant first exceeds four times its initial observation. [3]
	Since h is increasing, $\dfrac{80}{1+10\mathrm{e}^{-0.4t}}>\dfrac{4\times 80}{1+10\mathrm{e}^{0}}$ $1+10\mathrm{e}^{-0.4t}<\dfrac{11}{4}$ M1: correct set up of equation (accepts equal if students round up at the end, preferably with explanation that h is an increasing function) $\mathrm{e}^{-0.4t}<\dfrac{\frac{11}{4}-1}{10}=0.175$ M1: simplification to exponential eqn $-0.4t<\ln 0.175$ $t>4.3574$ (5 s.f.) $t=4.36$ (3 s.f.) A1 (accepts 5, though context need not be integer)

(c) The height, y cm, of another species of plant, t months after the first observation is given by $y = \frac{1}{a + be^{-t}}$, where a and b are constants. Explain clearly how a straight line graph can be drawn to represent this relationship. You should state which variables should be plotted on each axis and explain how the values of a and b can be calculated. [4]

$$y = \frac{1}{a + be^{-t}}$$

$$\frac{1}{y} = a + be^{-t}$$

M1: simplification to Y=mX + c form

Plot $\frac{1}{y}$ on the vertical axis against e^{-t} on the horizontal axis to obtain a straight line graph. [A1] The gradient of the graph would give the value of b [A1] and the y-intercept will give the value of a [A1].

Note: no award of full A1s if M1 not awarded.

9 (a) Without using a calculator, solve the equation $x\sqrt{15} + \sqrt{5} = x\sqrt{2} + \sqrt{6}$. Leave your answer in the form $p\sqrt{10} + q\sqrt{3}$, where p and q are fractions. [4]

$$x\sqrt{15} + \sqrt{5} = x\sqrt{2} + \sqrt{6}$$

$$x\left(\sqrt{15} - \sqrt{2}\right) = \sqrt{6} - \sqrt{5}$$

$$x = \frac{\sqrt{6} - \sqrt{5}}{\sqrt{15} - \sqrt{2}}$$

M1: $x = \frac{\sqrt{6} - \sqrt{5}}{\sqrt{15} - \sqrt{2}}$

$$= \frac{\sqrt{6} - \sqrt{5}}{\sqrt{15} - \sqrt{2}} \times \frac{\sqrt{15} + \sqrt{2}}{\sqrt{15} + \sqrt{2}}$$

M1: correct rationalisation fraction

$$= \frac{\sqrt{90} - \sqrt{10} - \sqrt{75} + \sqrt{12}}{15 - 2}$$

M1: expansion of numerator and denominator

$$= \frac{3\sqrt{10} - \sqrt{10} - 5\sqrt{3} + 2\sqrt{3}}{13}$$

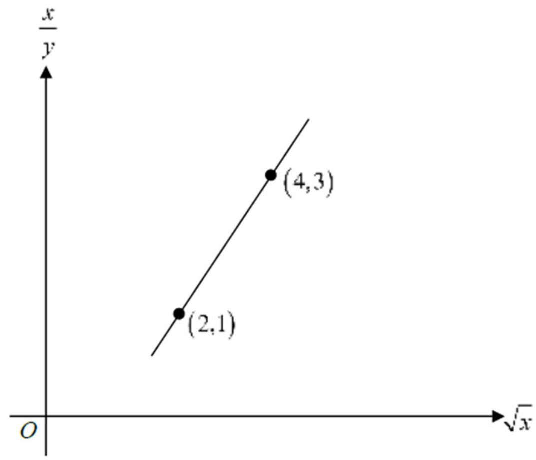
$$= \frac{2\sqrt{10} - 3\sqrt{3}}{13}$$

$$= \frac{2}{13}\sqrt{10} - \frac{3}{13}\sqrt{3}$$

A1: in $p\sqrt{10} + q\sqrt{3}$, where p and q are fractions

	(b) Without using a calculator, solve the equation $\log_2 x - \log_x 16 = 0$. [4]
	$\log_2 x - \log_x 16 = 0$ $\log_2 x - \dfrac{\log_2 16}{\log_2 x} = 0$ — M1: correct change of base $(\log_2 x)^2 = 4$ $\log_2 x = \pm 2$ — M1: simplification of $\log_2 16$ and reduced to a solvable quadratic equation $x = 2^2$ or $x = 2^{-2}$ $x = 4$ or $x = \dfrac{1}{4}$ — A1, A1
	(c) Solve the equation $3^{x+2} - 2(3^{-x}) = 17$. [4]
	$3^{x+2} - 2(3^{-x}) = 17$ $9(3^x) - \dfrac{2}{3^x} = 17$ $9(3^x)^2 - 17(3^x) - 2 = 0$ — M1: correct application of indices rules and rewriting into quadratic eqn in 3^x Let $u = 3^x$ $9u^2 - 17u - 2 = 0$ $(u-2)(9u+1) = 0$ — M1: factoring or formula to solve quadratic eqn $u = 2$ or $u = -\dfrac{1}{9}$ (rej since $3^x > 0$) — A1: Solving of 3^x with rejection $3^x = 2$ $x = \dfrac{\ln 2}{\ln 3} = 0.631$ (3 s.f.) — A1: do not accept $x = \log_3 2$ as no way of computing

10 **(a)** The diagram shows part of a straight line graph which passes through $(2,1)$ and $(4,3)$.



Find the equation of the straight line in the form $y = \dfrac{x}{a + b\sqrt{x}}$, where a and b are constants. [3]

gradient of straight line: $\dfrac{3-1}{4-2} = 1$

M1: gradient of line

equation:

$$Y - 1 = 1(X - 2)$$

$$Y = X - 1$$

M1: eqn of line

$$\frac{x}{y} = \sqrt{x} - 1$$

$$y = \frac{x}{\sqrt{x} - 1}$$

A1

(b) The table below shows the experimental values of two variables x and y.

x	1	2	3	4	5	6
y	63	127	258	510	1000	2100

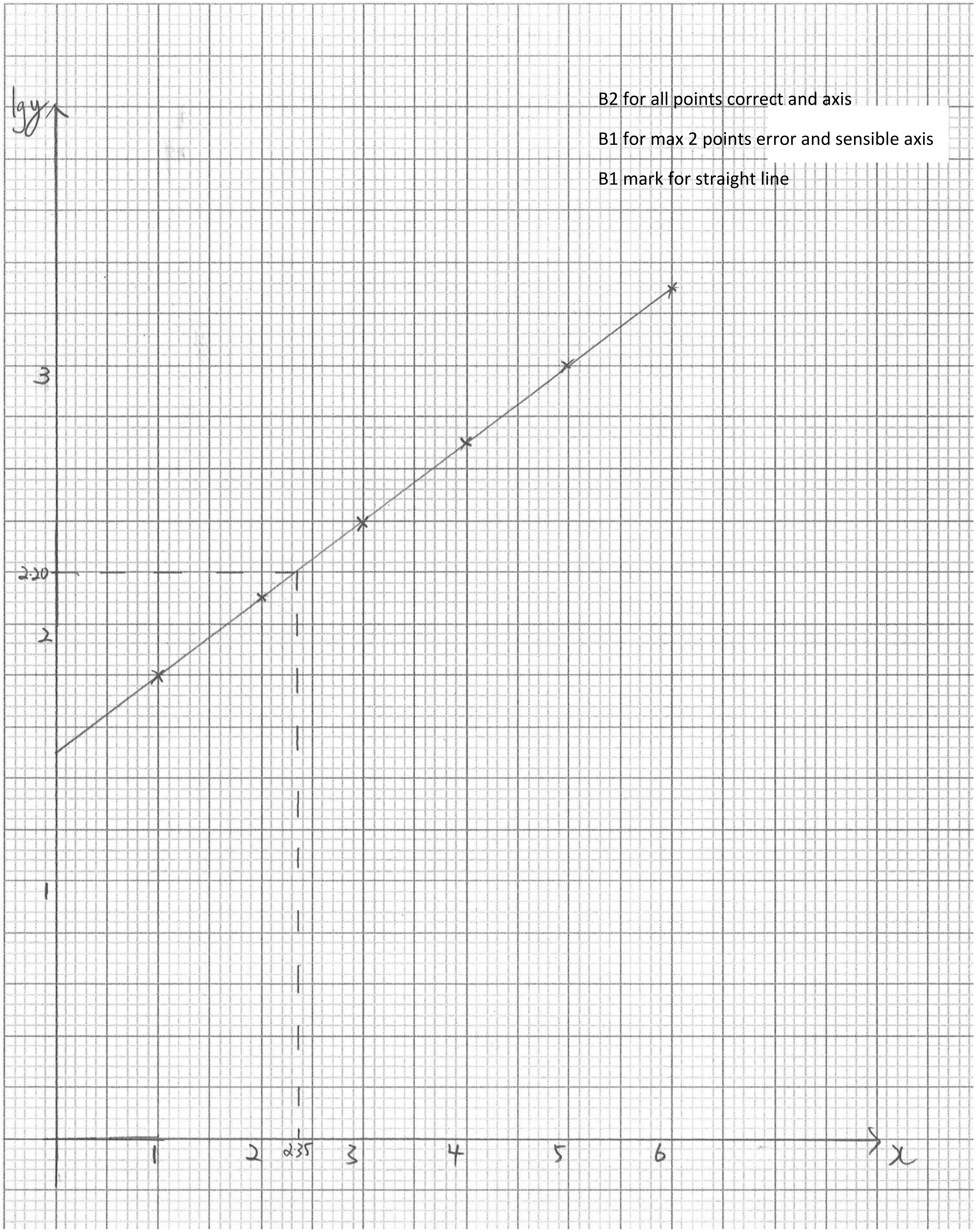
It is known that x and y are related by an equation of the form $y = \dfrac{b^x}{10^a}$, where a and b are constants.

(i) On the grid next page, plot $\lg y$ against x and draw a straight line graph. [3]

(ii) Use your graph to estimate the value of a and of b. [3]

$y = \dfrac{b^x}{10^a} \Rightarrow$

	$\lg y = \lg\left(\frac{b^x}{10^a}\right)$ $\lg y = \lg b^x - \lg 10^a$ $\lg y = x\lg b - a$ M1 $-a = 1.5$ $a = -1.5$ A1 (accept -1.54 to -1.46) $\lg b = 0.3$ $b = 2.00$ A1 (accept 1.82 to 2.19)
	(iii) Explain how you would use the graph to find the value of x for which $(10b)^x = 10^{a+1}$. [2]
	$(10b)^x = 10^{a+1}$ $10^x b^x = 10^a \times 10$ $\frac{b^x}{10^a} = \frac{10}{10^x}$ M1 $y = 10^{1-x}$ $\lg y = \lg\left(10^{1-x}\right)$ $\lg y = 1 - x$ Draw the line $\lg y = 1 - x$ and find the x-coordinate of the point of intersection. A1 OR $(10b)^x = 10^{a+1}$ $x\lg(10b) = (a+1)\lg(10)$ $x\lg(10) + x\lg(b) = a + 1$ $x\lg(b) - a = x + 1$ $\lg y = x + 1$



B2 for all points correct and axis

B1 for max 2 points error and sensible axis

B1 mark for straight line