Contents

Sec 2 Mathematics

Proportion

Chapter Summary

1. Map scale = Map length : Actual length

    $= 1 : n$ or $\dfrac{1}{n}$

2. Area scale = Area on a map : Actual area

    $= 1 : r^2$

3. When two quantities, X and Y, are in direct proportion, $Y = kX$, where k is a constant.

4. The graph of direct proportion ALWAYS pass through the origin.
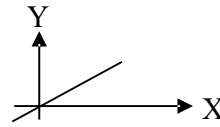
5. When two quantities, X and Y, are in inverse proportion, $Y = \dfrac{k}{X}$, where k is a constant.

6. The graph of Y against X is called a hyperbola.

7. Proportion problem sums formula :

    $$\frac{J_1}{n_1 \ t_1} = \frac{J_2}{n_2 \ t_2}$$

Proportion

Exam Questions:

Map Scales

1   A map of Singapore is drawn to a scale of 4 cm : 72 km.
  (a)   Express the scale of the map in the form 1 : n, where n is an integer.   [1]
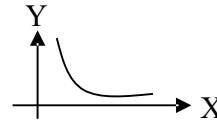 (b)   If the distance between Tiong Bahru and Tampines on the map is 9.5 cm, find the actual distance between the two towns in km.   [1]
  (c)   If the actual area of Tampines is 20.736 km^2, calculate its area on the map. [2]

2   Map A is drawn to a scale of 1 : 30 000.
  (a)   The area of a lake measures 50 cm^2 on Map A. Find the actual area of the lake in m^2.   [2]
  (b)   The length of a bridge on Map A is 6 cm long. Calculate the length of the same bridge, in cm, on Map B, given that Map B is drawn to a scale of 1 : 20 000.   [2]

3   A map is drawn to a scale of 1 : 600 000.
  (a)   Two points on the island are 0.7 cm apart on the map. Calculate the actual distance between these two points in metres.   [1]
  (b)   An island has an area of 28.8 km^2. Find the area that is represented on the map.   [2]
  (c)   On another map, the same island is represented by an area of 720 cm^2. Express the scale of the map in the form 1 : n.   [2]

4   An area of 324 km^2 is represented on a map by an area of 16 cm^2. Find
  (i)   the scale of the map in the form 1 : n.   [2]
  (ii)   the length of a road on the map with an actual distance of 81 km.   [2]

5   (i)   The scale of a map is 1 : n. An actual area of 72 m^2 is represented by an area of 32 cm^2 in the map. Find the value of n.   [2]

  (ii)   The length of a walking path is 10 m. Calculate the length, in centimetres, of the walking path on the map.   [2]

Direct and inverse Proportion (Basic)

1   R is directly proportional to the cube of p.
     When $p = 3$, $R = 54$.
     (a)   Find the formula for R in terms of p.   [2]
     (b)   Find the value of p when $R = 128$.   [1]
     (c)   Which of the diagrams below represents the graph of R against p
            for $p \geq 0$?   [1]

Diagram 1   Diagram 2   Diagram 3   Diagram 4   Diagram 5
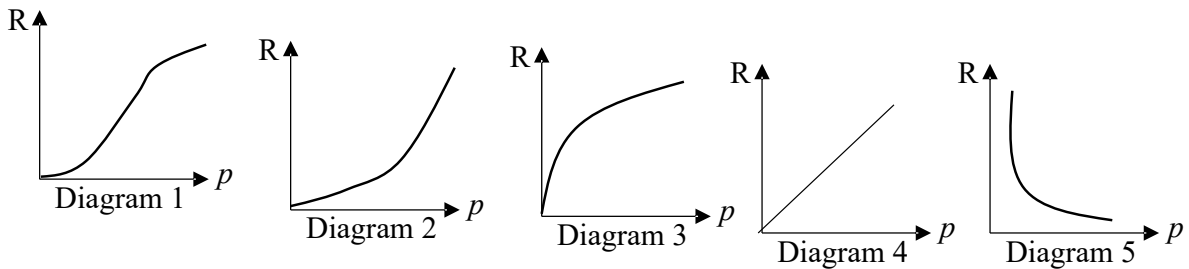
2   (a)   It is given that centrifugal force F is proportional to the square of the velocity,
            v, of the moving object. If F is 6250 N when the speed is 25 m/s, write down
            the formula relating F and v.   [2]
     (b)   Hence, find the value of F when the velocity of the object is increased by
            5 m/s.   [2]

3   The number of ball bearings, N, which can be made from a block of metal is inversely
     proportional to the cube of the radius, r mm, of each ball bearing. Given that $N = 280$
     when $r = 5$, find the value of r when $N = 4375$.

4   It is given that y is directly proportional to $x - 3n$. When $y = 20$, $x = 62$ and when
     $y = 35$, $x = 99.5$.
     (a)   Show that $n = 4$.   [3]
     (b)   Hence, express y in terms of x.   [2]

5   It is given that y is directly proportional to the cube root of x and that when
     $x = 8$, $y = 25$. Find the equation connecting x and y.   [2]

Direct and inverse Proportion (Intermediate)

1   N is directly proportional to x^2. When x has a certain value, $N = 8$. Find the value of N when x is halved.   [2]

2   The pressure, P, Pa, of an object is inversely proportional to the volume of a cube. The pressure is 1000 Pa when the length of the cube is l cm. Find the new pressure when the length of the cube increases by 400%.   [3]

3   It is given that x is directly proportional to y and y is directly proportional to z. Prove that $x + y$ is directly proportional to z.

4   A group of solid cones have equal heights. The volume, V cm^3, of each cone is directly proportional to the square of its radius, r cm. A cone with a particular radius has a volume of 260 cm^3. When this radius is increased by 50%, find the new volume of the cone.   [3]

5   Newtons law of universal gravitation states that a particle attracts every other particle in the universe using a force that is directly proportional to the product of their masses and inversely proportional to the square of the distance between their centres.
    When two particles of the same mass are at a distance of 10 m from their centres, the force is F newtons. Find the percentage change in the distance for these two particles, when the force is increased by 300%.   [3]

6   The magnetic force M between two magnets is inversely proportional to the square of the distance d between the two magnets.
    The difference in magnetic force is 15 N when the two magnets are placed 5 cm and 10 cm apart from each other.
    Find the magnetic force between the two magnets when they are 25 cm apart.   [3]

7   If y is inversely proportional to $(x - 1)$ and the difference in y when $x = 5$ and $x = 2.5$ is 1.5, find the value of y when $x = 0.25$.   [3]

Applications on Proportion

1   12 women take 5 hours to prepare 1600 gift packs. Given that all women work at the same rate, calculate the time taken, in hours, for 15 women to pack 2000 gift packs.

2   If 7 pumps take 2 hours to fill 100 tanks, how long would it take 5 pumps to fill 80 tanks, assuming each pump operates at the same rate? [2]

3   Xiao Ming and four of his friends were hired for a job. If Xiao Ming got one more friend to join the team, the job can be completed 6 days earlier. Assuming all of them work at the same rate, how many friends should Xiao Ming ask to join the original team so that they can complete the job 12 days earlier?

4   For an adventure camp, food is bought to last for 15 days for a group of 40 students. If 4 students left the camp after 6 days, how many days can the food last for the remaining 36 students? [3]

5   A developer estimates that he needs 50 workers to build a tower in 120 days. Assume that all the workers work at the same rate,
      (a)   If he has 20 workers, how long would he take to build the tower? [2]
      (b)   If the tower is to be completed in 100 days, find the additional number of workers needed to build the tower. [2]

Expansion and Factorisation

Chapter Summary

1. Expansion of $(a + b)(x + y) = ax + ay + bx + by$

2. $(a + b)^2 = (a)^2 + 2(a)(b) + (b)^2$

3. $(a - b)^2 = (a)^2 - 2(a)(b) + (b)^2$

4. $(a)^2 - (b)^2 = (a + b)(a - b)$

5. Factorisation of quadratic expression : $ax^2 + bx + c$
   Example : Factorise $x^2 - 5x + 6$

   Answer : $x^2 - 5x + 6 = (x - 2)(x - 3)$

6. Factorisation of two term expression :
   Example : Factorise $3x^2 - 75$
   $3x^2 - 75 = 3(x^2 - 25)$
   $\qquad = 3(x + 5)(x - 5)$

7. Factorisation of four term expression :
   Example : Factorise $ax - 3ay - 3by + bx$
   $ax - 3ay - 3by + bx = a(x - 3y) + b(\ 3y + x)$
   $\qquad\qquad\qquad = (x - 3y)(a + b)$

Expansion and Factorisation

Exam Questions:

Expansion

1   Expand and simplify $2(5 - 2x) - (3x - 1)(x - 4)$.

2   Simplify $4a^2 - (2a + 5)(2a - 5) + a$.

3   Simplify $5 - p(3p - 2) + (p - 1)(2p + 3)$.

4   (a)   Simplify $(x - y)(x + y) + y^2 + x^2$. [1]
     (b)   Hence, evaluate the value of $1950 \times 2050 + 2500 + 2000^2$ without using a
            calculator. [2]

5   Expand and simplify
     (a) $9xy - (0.5x + 3y)^2$ [2]
     (b) $(2x + 5y)(4x - y) - (x + 2y)(x - 2y)$ [3]

Identities

1   If $(x - y)^2 = 40$ and $-xy$ 5, find the value of $x^2 + y^2$.   [2]

2   If $(3a + 3b)^2 = 135$ and $2ab = 7$, find the value of $a^2 + b^2$.

3   Given that $(x - y)^2 = 5$ and $3xy = 12$,
     (i)   find the value of $x^2 + y^2$,
     (ii)   hence, find the value of $(3x - 5y)^2 - 16y^2$.

4   Given that $4b^2 - 4c^2 = 224$, $b + c = 7$ and $bc = \dfrac{15}{4}$,

     (i)   find the value of $b - c$,
     (ii)   find the value of $b^2 + c^2$.

5   Given that $x + y = 9$ and $xy = 7$, find the value of $(x - y)^2$.

6   By using a suitable algebraic identity, evaluate $213^2 - 48138 + 113^2$.   [3]

Factorisation(Two terms)

1   Factorise $9x^2 - 25$.   [1]

2   Factorise completely $5pq - 20p^3q$.

3   Factorise completely $8a^2 - 18$.

4   Factorise $(25a + 8b)(3a + 4b) - 7(3a + 4b)^2$ completely.   [2]

5   Factorise completely $9p^2 - (q - r)^2$.

Factorisation(Three terms)

1   Factorise $9x^2 + 30x + 25$ completely.   [1]

2   Factorise $2x^2 - 11x - 21$

3   Factorise the following completely.
$4x^2 - 8x - 12$ [2]

4   Factorise $6x^2 - 11xy - 10y^2$

5   Factorise $81m^2n - 90mnp + 25np^2$ completely.

Factorisation(Four terms)

1   Factorise $2n + 6 - mn - 3m$ completely.   [2]

2   (i)   Factorise completely $2ax + ay + 4bx + 2by$.   [1]
       (ii)   Hence, given that $2ax + ay + 4bx + 2by = 0$ when $a + 2b \neq 0$, find the
                value of $\dfrac{x}{y}$.   [2]

3   Factorise the following completely.
       $81k^2 - 16h^2 - (9k - 4h)$ [2]

4   Factorise $6ab - 2a - 3b^2 + b$.   [2]

5   (a)   Factorise completely $x^3 - 5x^2y - 4xy^2 + 20y^3$.   [3]
       (b)   Given that $x^3 - 5x^2y - 4xy^2 + 20y^3 = 0$, hence, find the values of $\dfrac{x}{y}$.   [2]

Simple Algebraic Fractions

Chapter Summary

Algebraic fractions

(a) Simplifying algebraic fractions : 1st case : $\dfrac{1 \text{ term}}{1 \text{ term}}$

 - cancel

Example : $\dfrac{15m^2n^2}{5m^4n^3} = \dfrac{3}{m^2n}$

 : 2nd case : $\dfrac{1 \text{ term}}{1 \text{ term}}$

 - factorise
 - cancel

Example : $\dfrac{3x-21}{7x-x^2} = \dfrac{3(x-7)}{x(x-7)} = \dfrac{3}{x}$

 : Special case

Example : $\dfrac{x-3}{x^2-x-6} = \dfrac{x-3}{(x-3)(x+2)} = \dfrac{1}{x+2}$

(b) Multiplication and Division of Algebraic fractions
 1st case :

Example : $\dfrac{6x^3y^2}{45y} \div \dfrac{18xy}{xy^2} \div 3y^3 = \dfrac{6x^3y^2}{45y} \cdot \dfrac{18xy}{xy^2} \cdot \dfrac{1}{3y^3} = \dfrac{4x^3}{5y^3}$

 2nd case :

Example : $\dfrac{6m-8n}{7pq+7pr} \div \dfrac{3m^2-4mn}{9qs+9rs} = \dfrac{2(3m-4n)}{7p(q+r)} \cdot \dfrac{9s(q+r)}{m(3m-4n)} = \dfrac{18s}{7pm}$

(c) Addition and subtraction of algebraic fractions
 1st case : Different denominators

Example : $\dfrac{4a}{3x} + \dfrac{5x}{7a} = \dfrac{28a^2+15x^2}{21ax}$

 2nd case : Common denominators

Example : $\dfrac{3}{5x-1} - \dfrac{4}{25x^2-1} = \dfrac{3}{5x-1} - \dfrac{4}{(5x+1)(5x-1)} = \dfrac{15x-1}{(5x+1)(5x-1)}$

Simple Algebraic Fractions

Exam Questions:

<u>Simplifying of fractions</u>

1   Simplify $\dfrac{4xy^3}{64xy^4}$

2   Simplify $\dfrac{2(3\quad x)^2}{12x\quad 4x^2}$

3   Simplify $\dfrac{-\ 5}{-3\quad +\ 5}.$ [3]

4   Simplify $\dfrac{3\ -3}{-\quad -\ +}.$ [3]

5   Simplify $\dfrac{c^2\quad 4d^2}{4bc + 7ac\quad 8bd + 14ad}.$

Multiplication and division of fractions

1   Simplify $\dfrac{3\ +}{-}\ \dfrac{}{+}$.   [3]

2   Express the following as a single fraction in its simplest form.

$$\dfrac{2x\ \ 3}{2x^2\ \ 5x+3}\ \dfrac{3\ \ 3x^2}{9}$$

3   Simplify $\dfrac{-4}{-3\ +}\ \dfrac{}{-}$.   [2]

4   Simplify $\dfrac{b^2\ \ 9}{20}\ \dfrac{2(b+3)}{b}$

5   Simplify $\dfrac{a+2}{a^2\ \ 2a+1}\ \dfrac{3a+6}{a\ \ 1}$

Addition and Subtraction of fractions

1   Express as a fraction in its simplest form

   (a) $\dfrac{2x+3}{2x-1} + \dfrac{x}{1-2x}$

   (b) $\dfrac{5}{(x+3)^2} - \dfrac{2}{x+3}$

2   Simplify $\dfrac{3}{3x^2-15x} - \dfrac{24x+18}{(9+12x)(5-x)}$.   [3]

3   Express $\dfrac{}{5-+3} - \dfrac{4}{5-3}$ as a single fraction in its simplest form.   [2]

4   Simplify $\dfrac{3}{-} - \dfrac{}{3-} - \dfrac{+}{-+3}$ as a single fraction in its simplest form.   [4]

5   Express the following as a single fraction in its simplest form.

   $2 - \dfrac{3x-7}{x^2-4x+4} + \dfrac{6}{x-2}$

Algebraic Manipulation and equations

Chapter Summary

1   A quadratic equation is expressed in the form
$ax^2 + bx + c = 0$, where a, b and c are constants and $a \neq 0$.

2   If $ax^2 + bx + c = 0$ can be written as
$(px + q)(rx + s) = 0$,

then $x = \dfrac{q}{p}$ or $x = \dfrac{s}{r}$.

3   Changing the subject of the formula

(a)   Basic

     Example : Make x the subject of the formula $y = 2mx + b$

$$x = \frac{y \quad b}{2m}$$

(b)   Intermediate
     Cross multiple rule

     Example : Make x the subject of the formula

$$y = \frac{3x \quad 2y}{w \quad 4x}$$
$$wy \quad 4xy = 3x - 2y$$
$$3x + 4xy = wy + 2y$$
$$x(3 + 4y) = wy + 2y$$
$$x = \frac{y(w + 2)}{3 + 4y}$$

(c)   Intermediate
     Reciprocal rule

     Example : Make x the subject of the formula

$$\frac{a}{x} + \frac{y}{b} = 1$$
$$\frac{a}{x} = 1 \quad \frac{y}{b}$$
$$\frac{a}{x} = \frac{b \quad y}{b}$$
$$\frac{x}{a} = \frac{b}{b \quad y}$$
$$x = \frac{ab}{b \quad y}$$

(d)   Intermediate
     Radical form

     Example : Make x the subject of the formula

$$s = \frac{x^2 \quad u^2}{2a}$$
$$2as = x^2 \quad u^2$$
$$x^2 = 2as + u^2$$
$$x = \sqrt{2as + u^2}$$

Algebraic Manipulation and equations

Exam Questions:

<u>**Equations**</u>

1   Solve $\dfrac{x}{3} + \dfrac{2}{x+1} = 2$

2   (a)   Solve the quadratic equation $6x^2 + x - 15 = 0$.   [2]

         (b)   There were x students in a class. Each student in the class bought a present for each of their classmates and their two form teachers during their graduation ceremony. A total of 1640 presents were bought. Write down, but do not evaluate, a quadratic equation in the form of $ax^2 + bx + c = 0$, where a, b and c are constants, to represent the information.   [2]

3   Solve $(5x + 1)(x - 3) = 18 - (1 - x)$.   [3]

4   Solve $\dfrac{}{-3} \; \dfrac{}{3-} \; \dfrac{}{-} \; 3.$ [4]

Changing the subject of the formula (Basic)

1   It is given that $x = \dfrac{1}{5}yz^2$.

     (i)   Find x when $y = 5$ and $z = 4$.

     (ii)   Express z in terms of x and y.

2   (i)   Make q the subject of the formula $p = q^2 - 3$.

     (ii)   Hence, find the values of q given that $p = 46$.

Changing the subject of the formula (Two subject)

1   Express x in terms of a, b and c, given that $b + a = \dfrac{2x + c}{x}$

2   Given that $\dfrac{3}{5}$ $\dfrac{4-}{4+}$, express z in terms of x and y.   [4]

3   Given that $\dfrac{3-}{-5}$ $-$, express c in terms of a and b.   [2]

Changing the subject of the formula (Fractional equations)

1   Given $-\ \ -\ \ \dfrac{5}{3}$, make x the subject of the formula.   [3]

1   Given $-\ \ -\ \ \dfrac{5}{3}$, make x the subject of the formula.   [3]

Changing the subject of the formula (Radical)

1 Given that $x = \sqrt{\dfrac{5\ y^2}{y^2}}$,

    (i)   find the value of x when $y = 1$,

    (ii)   express y in terms of x.

2 (a) Make k the subject of the formula $p\quad r\quad \overline{}_{+}$.   [2]

    (b) Hence, find the value of k when $p = 5$, $q = 2$ and $r = 3$.   [1]

3 Given that $T = 2\sqrt{\dfrac{c\ 1}{2a}}$,

    (a) make a the subject of the formula.   [3]

    (b) Hence, find the value of a when $T = 3$ and $c = 8$.   [2]

4 Given that $k = m\sqrt[3]{3 + \dfrac{2k}{m^3}}$, express m in terms of k.   [3]

Linear Equations in one variable

Chapter Summary

1   Simultaneous Equations

It can be solved by either (a)   Elimination method
                          (b)   Substitution method
                          (c)   Graphical method

(a)   Elimination method

Make the coefficient of one of the variable the same for both equations and then either add or subtract both equations to form a linear equation with only one unknown.

Example : $3x + y = 4$ -- (1)
          $4x + 2y = 5$ -- (2)

(1) × 2 : $6x + 2y = 8$-- (3)
(3) – (2) : $2x = 3$
            $x = 1.5$
Sub x into (1) : $3(1.5) + y = 4$
            $y = 0.5$

(b)   Substitution method

Make one of the variables the subject of an equation and then substitute it into the other equation.

Example : $3x + y = 4$ -- (1)
          $4x + 2y = 5$ -- (2)

From (1), $y = 4 – 3x$
Sub y into (2) : $4x + 2(4 – 3x) = 5$
            $4x + 8 – 6x = 5$
              $2x = 3$
            $x = 1.5$
Sub x into y : $y = 4 – 3(1.5)$
            $y = 0.5$

Linear Equations in one variable

Exam Questions:

<u>Solving through algebraic means</u>

1.   Solve the simultaneous equations.
     $3x = 2y + 3$
     $4x - 6y = 7$ [3]

2.   Solve the simultaneous equations:
     $3x - 2y - 13 = 0$
     $5x + 3y = 9$ [3]

3.   Solve the simultaneous equations
     $2x - y = 10$

     $x \quad - \quad \dfrac{}{3}$ [3]

4.   Solve the following pair of simultaneous equations.

     $-x \quad \dfrac{}{3}y \quad 4$

     $-x \quad -y \qquad 4-$ [3]

5.   Solve the simultaneous equations
     $\dfrac{3}{x} + \dfrac{5}{y} = 8$
     $\dfrac{4}{x} + \dfrac{13}{y} = 2$

Solving through graphical means

1 The graph of $y = x + 1$ is drawn on the axes below.

(a) The table below shows some corresponding values of x and y for the equation $y = \dfrac{1}{2}x + 4$. Complete the table.

x	2	0	4
y	5		

(b) On the axes above, draw the graph of $y = \dfrac{1}{2}x + 4$.

(c) Use your graph to solve the simultaneous equations.
$y = x + 1$
$y = \dfrac{1}{2}x + 4$

Problem Sums

Chapter Summary

1   Quadratic equation Problem Sums

(a)   Read the problem carefully.
(b)   Let x be an unknown quantity
(c)   Express some other quantities in terms of x.
(d)   Set up an equation of x using the given information.
(e)   Solve the equation.
(f)   Check the solutions.

2   Simultaneous equation Problem Sums

(a)   Read the problem carefully
(b)   Identify and define the two unknown quantities in the context.
(c)   Use variables, say x and y, to represent the two unknown quantities
(d)   Identify the relationships between x and y using the information given in the context.
(e)   Form the relationships using two linear equations.
(f)   Solve the simultaneous equations.
(g)   Check if the solution makes sense by referring them back to the context.

Problem Sums

Exam Questions:

Simultaneous Problem Sums

1   At a pet shop, James bought 5 guinea pigs and 5 swordfishes for $100 while Shirley
    bought 10 guinea pigs and 17 swordfishes for $287.50. Let x represents the price of a
    guinea pig and y be the price of a swordfish.
    (a)   Form two equations in terms of x and y.
    (b)   Calculate the total cost of 3 guinea pigs and 2 swordfishes.

2   The diagram shows a triangle ABC of height 3 cm. AC = $(5x + y)$ cm and
    BC = $(3x - 4y + 8)$ cm. The ratio of AC : BC is 2 : 3.
    (a)   Write down an equation, in terms of x and y, and show that it simplifies to
          $9x + 11y = 16$.
    (b)   The area of triangle ABC is 31.5 cm^2. Write down another equation, in terms
          of x and y, and simplify it.
    (c)   Solve these equations to find the value of x and the value of y.
    (d)   Hence, find the length of BC.
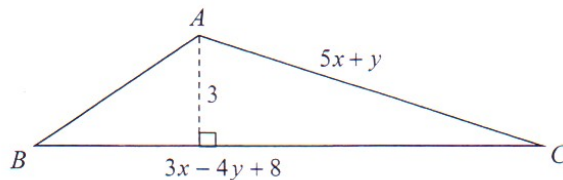
3   The diagram shows part of the price list at a fruit store.

Orange …….. 60 cents each
Dragonfruit ….$1.10 each

    Stefanie bought p oranges and q dragonfruits and paid $22.30. Tanya bought
    q oranges and p dragonfruits and paid $3 more than Stefanie.
    (a)   Write down two simultaneous equations, in p and q, to represent this
          information.
    (b)   Solve the simultaneous equations to find p and q.

4   The frequency table shows the number of books borrowed by 50 students from a library in a day.

Number of books	3	4	5	6
Frequency	20	p	14	q

(i)   Show that $p + q = 16$,
(ii) Given that the mean number of books borrowed is 4.4, show that $2p + 3q = 45$.
(iii) Hence, find the value of p and q.

5   The sum of the digits of a two-digit number is 13. When the digits of the number are reversed and the resulting number is subtracted from the original number, the result obtained is 45. By letting x to be the tens digit and y to be the ones digit, find the original number by forming a pair of simultaneous equations and solving them.   [4]

Quadratic Problem Sums

1   During the peak season, Ben paid $1680 as entrance fees for x tickets to Universal Studios Japan.
   (a)   Write down an expression, in terms of x, for the cost of a ticket in dollars.   [1]
   (b)   Had Ben gone during the off-peak season, he could buy 20 more tickets with the same amount of money, write down an expression, in terms of x, for the cost of a ticket in dollars during the off peak season.   [1]
   (c)   Given that the cost of a ticket during the off peak season is $25 cheaper, form an equation in x and show that it reduces to $x^2 + 20x - 1344 = 0$.   [3]
   (d)   Solve the equation and find the number of tickets Ben could have bought had he gone during the off-peak period.   [3]
   (e)   The Universal Studios Japan intend to give senior citizens a 40% discount on the price of the ticket during off peak season. How much must senior citizens pay for a ticket during the off peak season?   [1]

2   A photocopier print pages in either "black and white or in colour.
   (a)   In one minute, it prints x pages in black and white. Write down an expression, in terms of x, for the number of seconds, it takes to print one page in black and white.
   (b)   In one minute, it prints 2 more copies in black and white than it does in colour. Write down an expression, in terms of x, for
          (i)   the number of pages it prints in colour in one minute,
          (ii)   the number of seconds it takes to print one page in colour.
   (c)   It takes 1.5 seconds longer to print one page in colour than it takes to print one page in black and white.
          (i)   Write down an equation in x to represent this information, and show that it reduces to $x^2 - 2x - 80 = 0$.
          (ii)   Solve the equation $x^2 - 2x - 80 = 0$.
          (iii) Find the time taken in minutes and seconds to print 30 coloured pages.

3   A rectangular tank has a base of 3 m by 2 m and a height of 5 m. For the first 3 minutes, Tap A was used to fill up the tank. Thereafter, Tap B was introduced to hasten the filling up process. The tank was subsequently filled up to the brim in another $(5x - 0.5)$ minutes by using both Tap A and Tap B.
        Given that the rate of flow of water for Tap A and Tap B are x m^3/min and $(2x + 3)$ m^3/min respectively, write down an expression in terms of x, for the total volume of water in the rectangular tank that flowed from
   (a)   (i)   Tap A,   [1]
          (ii)   Tap B.   [1]
   (b)   Form an equation in x and show that it can be reduced to $10x^2 + 11x - 21 = 0$.   [2]
   (c)   Solve the equation $10x^2 + 11x - 21 = 0$ to find the percentage of water in the tank that came from Tap B.   [3]

4   A delivery truck travels its entire 80 km journey at an average speed of x km/h.

(a)   Write down an expression, in terms of x, for the time taken, in hours, for its
journey.   [1]

(b)   One day, the delivery truck was delayed due to heavy traffic and it took
30 minutes longer than schedule to travel the first 20 km. The remainder of the
journey was completed at an average speed of $(x + 20)$ km/h. Write down an
expression, in terms of x, for the time taken, in hours, to travel
(i)   the first 20 km,   [1]
(ii)   the remaining journey.   [1]

(c)   On the day with heavy traffic, the delivery truck still managed to arrive at its
destination on time.
(i)   Write down an equation in x, and show that it reduces to
$x^2 + 20x - 2400 = 0$.   [3]
(ii)   Solve the equation $x^2 + 20x - 2400 = 0$.   [2]
(iii)   Find the time taken for the normal scheduled journey.   [1]

5   (a)   Jack bought $400 of durians at the price of $$d$ per kg. Write in terms of d, an
expression for the number of kilograms of durian that Jack bought.   [1]
(b)   Jack sold the durians at $6 per kg more than the cost price and he collected
$658. Write down an expression, in terms of d, for the number of kilograms of
durian that Jack sold.   [1]
(c)   Given that his wife kept 3 kg of the durians, write down an equation in d to
relate this information, and show that it reduces to $d^2 + 92d - 800 = 0$.   [2]
(d)   Solve the equation $d^2 + 92d - 800 = 0$.   [2]
(e)   Hence, calculate the number of kilograms of durian Jack bought.   [1]

Congruence and Similarity

Chapter Summary

1.   Two figures are congruent if they have the same shape and size.
      (i.e. all corresponding angles and sides are equal)

2. $\triangle ABC \quad \triangle PQR$
      Angle A = Angle P
      Angle B = Angle Q
      Angle C = Angle R
      AB = PQ
      BC = QR
      AC = PR

3.   Two figures are similar if they have the same shape but not necessarily the same size.
      (i.e. all corresponding angles and ratios of their corresponding sides are equal)

4. $\triangle ABC$ and $\triangle PQR$ are similar.
      Angle A = Angle P
      Angle B = Angle Q
      Angle C = Angle R
      $$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$$

5.   Scale factor $= \dfrac{\text{image length}}{\text{object length}}$

Congruence and Similarity

Exam Questions:

Congruent figures

1.   In the diagram below, $\triangle$ABC is congruent to $\triangle$PRQ. It is given that BC = 29 cm, PQ = 20 cm, $ABC = 43.6^o$ and $CAB = QPR = 90^o$.

Find
(a)   the length of AB,
(b)   the size of PQR.

2.   In the diagram, triangle PQR is congruent to triangle QST. Given that PQ = 12 cm, ST = 8 cm and angle PQR = 40º, find
(a)   the length of RS, and
(b)   angle QTS.

3   In the diagram below, ΔABC is an isosceles triangle with $ADC = 90^{o}$ and
     $DAC = 48^{o}$.
     (a)   Name a pair of congruent triangles.
     (b)   State the reasons for the congruence.
     (c)   Find ABD

4   In the figure below, ABCDEF is part of a regular polygon.
     (a)   Name two triangles that are congruent.
     (b)   Given that angle EFD = 15°, find angle ABD.

5   The figure below consists of two overlapping congruent quadrilaterals.
     Given that $\angle AFC = 62°$, $\angle HDC = 103°$ and $\angle BCF = 90°$,

(a)   state the quadrilateral that is congruent to ABCF.   [1]
(b)   find the value of
       (i)   *x*,   [2]
       (ii)   *y*.   [1]

Similar figures

1   ΔABC is similar to ΔADE. Find *x* and *y*.

2   In the diagram, triangle ABX is similar to triangle CDX.
     Angle ABX = angle CDX. All measurements are in centimetres.
     Calculate
     (a)   CD,   [1]
     (b)   AD.   [2]

3 In the diagram, triangle WXZ is similar to triangle YWZ. All measurements are in centimetres.

(i)   Given that WZ = (x + 6) cm and XZ = (4x – 2) cm, find the value of x.   [3]

(ii) Find $\dfrac{\Delta}{\Delta}$ [2]

4 In the diagram, AC = 11 cm, BC = 8 cm, DE = 5 cm and BF = 7 cm. BF is perpendicular to AC. Triangle AED is similar to triangle ABC.

(a)   State the corresponding angle of angle ABC.

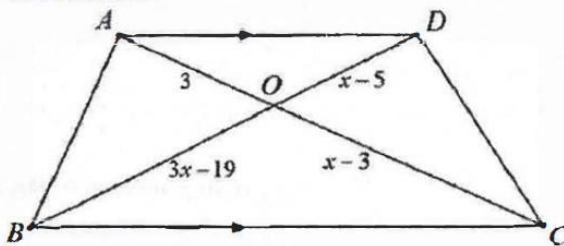(b)   Calculate the length of AD.

(c)   Calculate the length of AF.

5   In triangle ACD, B and E are points on AC and AD respectively such that
     triangle ECD is congruent to triangle BDC. The lines EC and BD intersect at P where
     PE = PB and DE = 25 cm, DC = 42 cm. The heights of triangle PEB and triangle PDC
     are 9.6 cm and 14.4 cm respectively.
     (a)   Explain why EB is parallel to DC, stating your reasons clearly.   [1]
     (b)   It is given that triangle AEB is similar to triangle ADC. State another pair of
            similar triangles that are not congruent.   [1]
     (c)   Hence or otherwise, show that EB = 28 cm.   [2]
     (d)   Calculate the length of AE.   [2]
     (e)   Calculate the area of triangle AEB.   [2]

6   ABCD is a trapezium in which AD is parallel to BC.
     The diagonals AC and BD intersect at O.

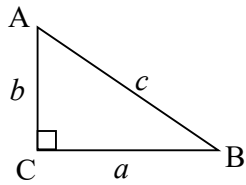   (a)   Name the triangle which is similar to $\triangle$AOD.   [1]

     (b)   If AO = 3 cm, BO = (3x – 19) cm, CO = (x – 3) cm and DO = (x – 5) cm, find
                                                                                  [3]

ABCD is a trapezium in which AD is parallel to BC.
The diagonals AC and BD intersect at O.

(a)   Name the triangle which is similar to $\triangle AOD$.

                                 Answer (a) Triangle _____ [1]
(b)   If AO = 3 cm, BO = (3x – 19) cm, CO = (x – 3) cm and DO = (x – 5) cm, find the possible
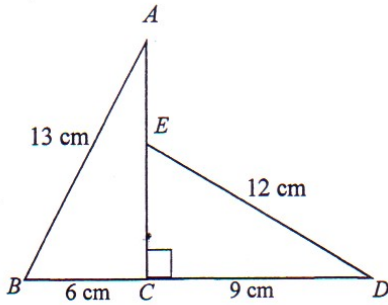   values of x.

Pythagaros Theoreom

Chapter Summary

1   In $\triangle$ABC, if $C = 90^o$, then $a^2 + b^2 = c^2$.
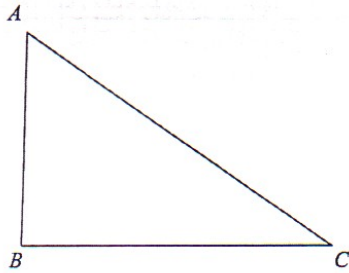

2   In $\triangle$ABC, if $a^2 + b^2 = c^2$, then $C = 90^o$.

Exam Questions:
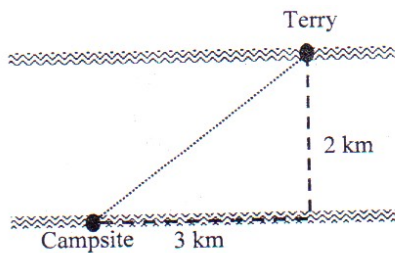
1   (a)   In the diagram below not drawn to scale, BCD is a straight line and is perpendicular to AEC. If AB = 13 cm, DE = 12 cm, BC = 6 cm and CD = 9 cm, find the length AE.

    (b)   In triangle ABC, AB = 8 cm, BC = 15 cm and AC = 18 cm. Determine whether ABC is a right angle. Show your working clearly.
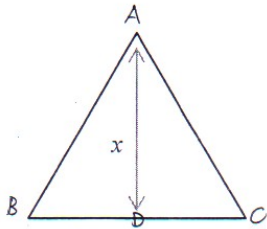

2   Terry is standing on the shore of a 2km wide river. He wants to reach a campsite that is 3 km down shore on the opposite side of the lake.
Terry can swim at a speed of 3 km/h and walk at a speed of 5 km/h. Showing your workings clearly, determine whether it is faster for him to swim perpendicularly across the river then walk down shore to the campsite or swim directly to the campsite.

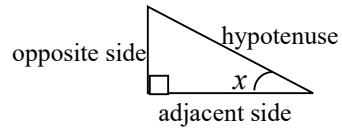3   In triangle ABC, AB = 85 cm, BC = 80 cm and AC = 36 cm. Explain why triangle ABC is not a right-angled triangle.

4   The perimeter of an equilateral triangle is 264 cm. Find the perpendicular height, x cm, of the triangle.

Trigonometry

Chapter Summary

1 Trigonometric Ratios in a Right-angled triangle

opposite side · hypotenuse · x · adjacent side

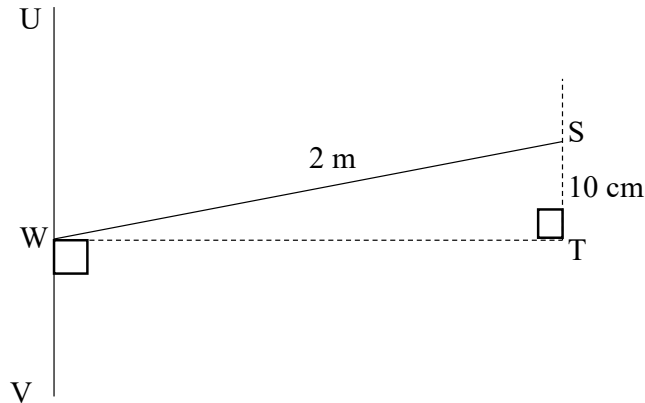$$\sin x = \frac{\text{opposite side}}{\text{hypotenuse}} \qquad \cos x = \frac{\text{adjacent side}}{\text{hypotenuse}} \qquad \tan x = \frac{\text{opposite side}}{\text{adjacent side}}$$

Trigonometry

Exam Questions:

1 The bamboo sticks, used for drying clothes in some HDB flats, are usually fixed at an angle to prevent the wet clothes from slipping off the sticks. UWV is the wall, WS is the bamboo stick of 2 m long and WT is perpendicular to the wall.

(a) Calculate the value of cos ∢TSW, expressing your answer as a fraction. [1]
(b) State the angle(s) in the diagram that has the same sine ratio as ∢TSW. [1]
(c) Find the length of WT, to the nearest 4 significant figures. [1]
(d) Evaluate ∢SWV. [1]

2 Victoria participates in a kite flying event at Marina Bay. She stands at a position, E, on the ground, and releases 200 m of string to fly the kite to position F. The string makes an angle of 40° with the ground.
(a) How high is the kite above the ground assuming that the string is taut?
(b) The wind becomes stronger and the kite is carried 60 m further away from point F to point G. Victoria releases more string, maintaining the height of the kite above the ground.
(i) Calculate the length of the string EG.
(ii) Calculate the angle that the string EG now makes with the ground.
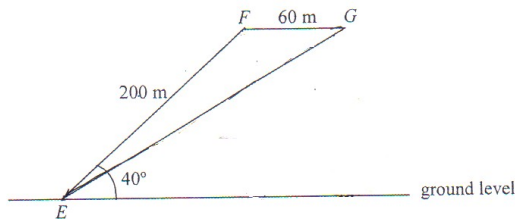
3   The diagram shows a triangle ABD and C is a point on BD such that AC = 17 cm and CD = 10 cm.

A

17 cm

B   C   10 cm   D
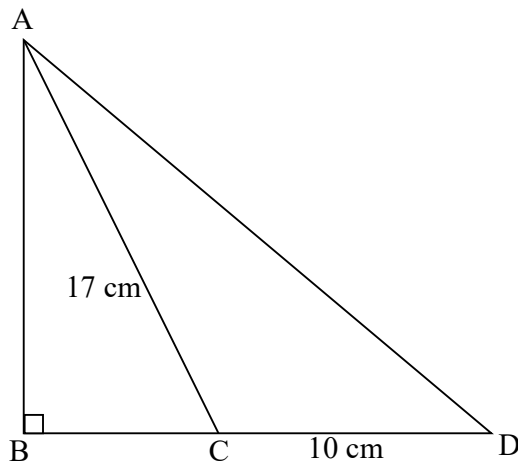
Given that angle ABD = 90° and sin $ACB = \dfrac{15}{17}$, find

(a)   the length of BC,   [2]
(b)   the area of triangle ACD,   [2]
(c)   tan ADB, leaving your answer as a fraction in its simplest form.   [1]

4   Jerry is riding in a cable car A moving towards the cable car station at T. He spotted a cat at the base of the station, point C, at an angle of depression of 19.5°. The height of the cable car station, TC, is 70 m and the distance between cable car B and the cat is 98 m.
(a)   Find the distance between the two cable cars A and B.
(b)   Find the angle of elevation of cable car B from the cat C.
(c)   Given that the Building and Construction Authority requires the safety distance between two cable cars must be at least 60 m, find the number of additional cable cars that can be added between AT, without compromising the safety of the passengers. Justify your decisions with calculations.

A   19.5°   B   T

98 m

70 m

C
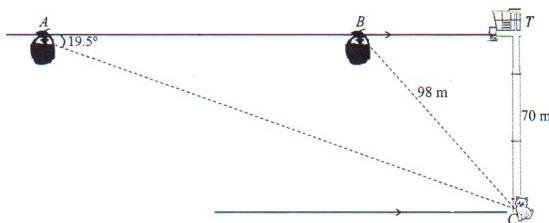
5   Evan went on a hot-air balloon ride starting at A. When Evan was vertically below B, the angle of elevation from A to the hot-air balloon was 68°. B is 250 m away from A. A vertical observatory tower is at C. The tower is 230 m high and 1.2 km away from A.

(a)   Find the height of the hot-air balloon when it as vertically above B.   [2]

(b)   A tourist spotted the hot-air balloon from the top of the observatory tower. Find the angle of elevation from the tourist to the hot-air balloon.   [3]
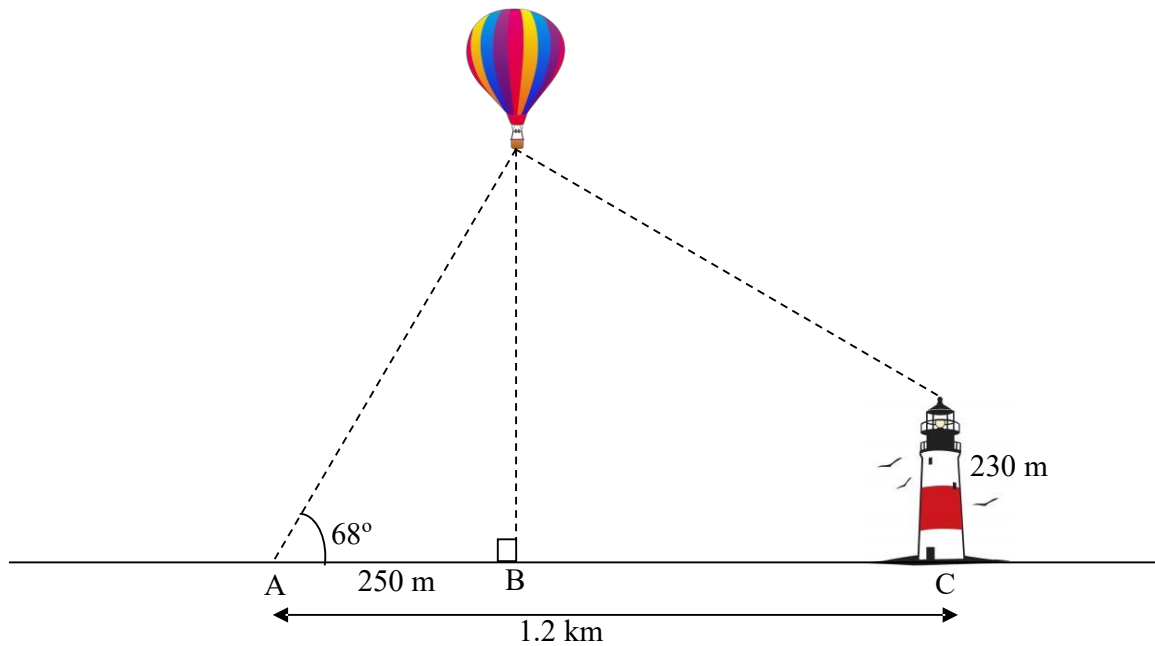
6   In the figure ABCDEF below, ∡ABC = ∡AED = ∡CDE = 90°. CF is parallel to DE.

(a)   Express AC in terms of AB.   [1]

(b)   Given that DE is twice of AB, find ∡ACF.   [2]
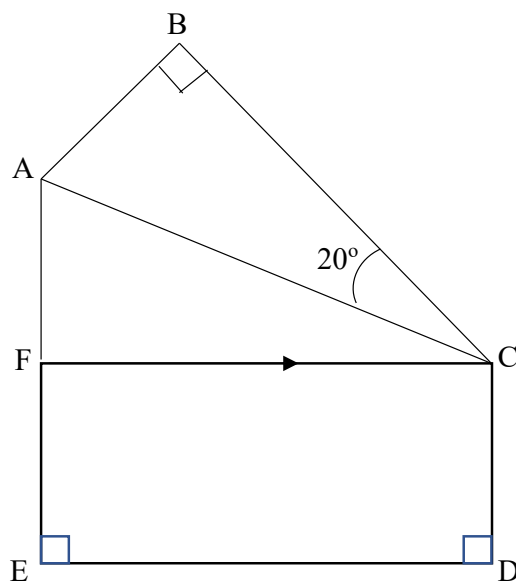
Mensuration

Chapter Summary

1. PYRAMID
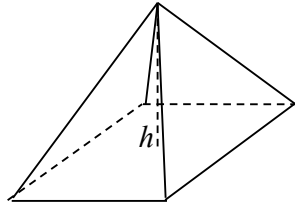
Total surface area of a pyramid = base area + area of all lateral faces

Volume of a pyramid = $\frac{1}{3}$ base area $\times$ height

2. CONE

Curved surface area of a cone = $\pi r l$

Total surface area of a cone = $\pi r l + \pi r^2$

Volume of a cone = $\frac{1}{3} \pi r^2 h$
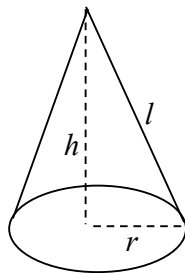
3. SPHERE

Surface area of a sphere = $4 \pi r^2$

Volume of a sphere = $\frac{4}{3} \pi r^3$
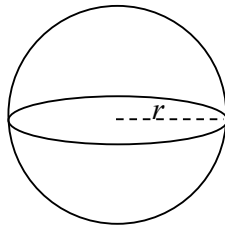
Mensuration

Exam Questions:

<u>Pyramid</u>

1   Eight pieces of wire, each 10 cm long, are tied together to form a square pyramid.
     Calculate the volume of the space enclosed by the pyramid.   [4]
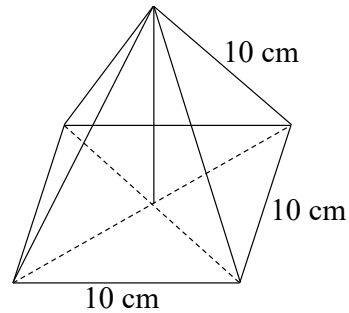
2   The diagram below shows an upright rectangular pyramid. AB = 8 cm, AD = 6 cm
     and the slant height EG = 13 cm. Find
     (a)   the height of the pyramid EF,
     (b)   the volume of the pyramid.
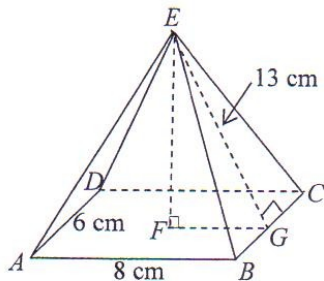
3   (a)   Find the volume of the square pyramid, as shown in the diagram below, whose height is 12 cm and its square base has sides of 18 cm.
    (b)   Find the total surface area of the square pyramid.

12 cm

18 cm

18 cm
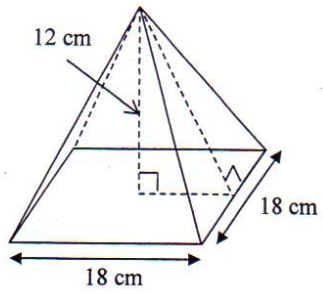
Cone

1 The diagram shows a circle with centre O with one quarter of it cut off. A open cone is formed by joining the two edges OA and OB.

(i) The surface area of the cone is 12 cm^2. Find the value of x. [2]

(ii) Find the radius of the cone. [2]
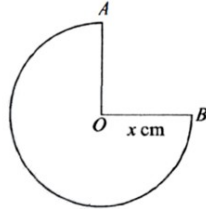
2 A A fixture at the playground may be modelled by a frustum, *ABPQ*, as shown, PQ, as shown. This is
obt This is obtained by removing the smaller cone *OAB* from the larger cone, ne OPQ.
OPQ.
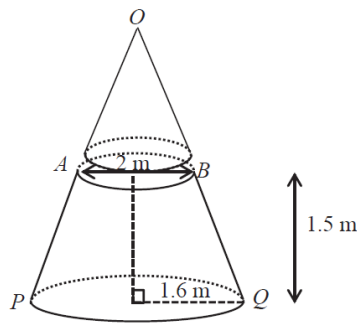
The base diameter of the cone OAB is 2 m. The base radius of cone OPQ is 1.6 m and the height of the frustum is 1.5 m.

(a) (i) Show that the height of the cone OAB is 2.5 m. [2]

(ii) Hence, find the volume of the fixture, ABPQ. [2]

(b) (i) The solid is melted and recast to form a solid cylinder with a height of 3 m. Find the radius of the cylinder. [2]

(ii) Each tin of paint is enough to paint an area of 60 m^2. Find the number of tins of paint required to paint 500 identical cylinders. [2]

3   The diagram shows an open container made from a cone and a cylinder. The cone has radius r cm, slant height l cm and height 12 cm. The cylinder has the same base radius and height 20 cm. The volume of the conical part of the container is 100 cm^3.

(a)   (i)   Show that the value of r is 5.

(ii)   Hence, calculate the total exterior surface area of the open container.

(b)   1 litre of water is poured into the empty container. Calculate the height of the water in the container.
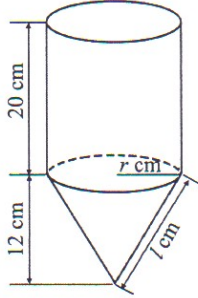
Spheres

1   90 spheres each of radius 1.5 cm are packed into a rectangular wooden box. The first diagram shows the front cross-sectional view of the box with
6 spheres in one layer. The second diagram shows a 3-dimensional view.
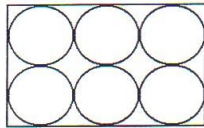
Front Cross-sectional view
of box with 6 spheres in the
first layer
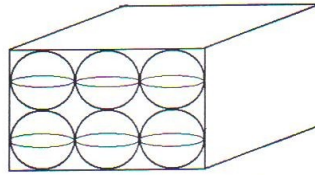
3-D view of 90 spheres inside a
rectangular box

Calculate
(i)   the volume of 90 spheres,
(ii)   the volume of the inside of the box which is
        unoccupied by the spheres,
(iii) The 90 spheres are to be repacked into cylindrical containers. Each container consists of three spheres packed tightly together. Assuming that the same material is used for both the rectangular and cylindrical boxes, which is the more economical way to pack the 90 spheres? Show your working clearly to justify your conclusion.

2   A hemisphere of diameter 7 cm is removed from a hemispherical solid of diameter 14 cm as shown in the figure below. Find the total surface area of the remaining solid. Leave your answer in terms of .
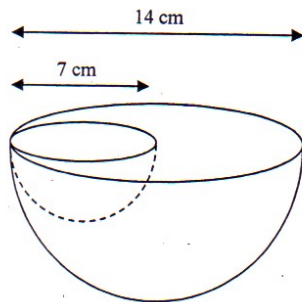
3   In the current modern days, people are seeking quality handmade products over mass-manufactured ones which could be environmentally unfriendly.
Figure 1 shows a handmade wooden spoon made from a block of wood and **Figure 2** shows an example of a handmade wooden spoon.

Figure 1

Figure 2

Diagram 1 shows the wooden spoon modelled after a hemispheric bowl and a long cylindrical handle.
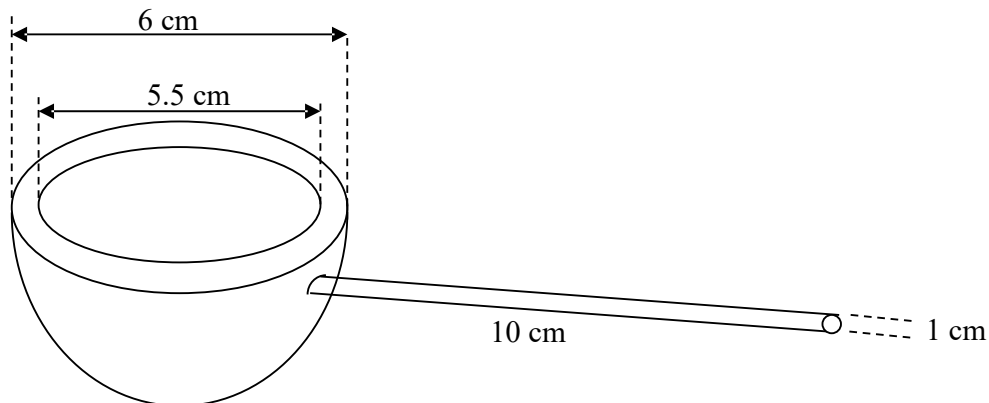
Diagram 1

Assuming that the hemispheric bowl has equal thickness throughout, the external and the internal diameter is given by 6 cm and 5.5 cm respectively.
The long cylindrical handle has a diameter of 1 cm and an estimated length of 10 cm.

(a)   (i)   Calculate an estimate for the volume of the wooden spoon.   [3]

(ii)   Explain why the volume of the wooden spoon in part (a)(i) can only be an estimate.   [1]

(b)   Wooden products need to be coated with a layer of wood balm often to protect the wood. Given that a 50 ml bottle of wood balm can coat 4000 cm^2 of surface area, calculate and estimate the maximum number of spoons that can be coated with a bottle of wood balm.   [4]

(e)   The entire piece of wooden spoon is carved from a single rectangular block of sycamore wood.

(i)   State the minimum volume for the rectangular block of sycamore wood needed to carve out each wooden spoon, showing your workings clearly.   [1]

(ii)   Hence, find the minimum total raw material cost for making each wooden spoon, given that sycamore wood cost $0.15 per cm^3.   [1]

Misc

1   Mr Ho, an ice cream vendor, sells biscuit cone filled with soft ice cream and name it
     as Crispy ice cream.
     He uses biscuit cone in Figure 1, which can be modelled as a cone as shown in
     Figure 2.
     The height of the cone is 10 cm and its radius is 3.2 cm.
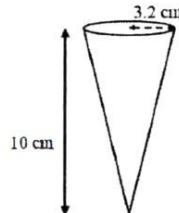
Figure 1   Figure 2

Mr Ho always fills the biscuit cone completely with soft ice cream. After which he
tops up with an amount of ice cream to form a hemisphere at the top, as shown in
Figure 3 below.

Figure 3

(a)   Assuming that the thickness of the cone is negligible, calculate the total
       volume of ice cream used by Mr Ho for one Crispy ice cream.   [2]

(b)   The cost price of one Crispy ice cream includes the cost of the biscuit cone
       and the ice cream. The cost price of an empty biscuit cone is 15 cents and the
       cost price of 10 cm^3 of ice cream is 5 cents. Calculate the cost price, in cents,
       of one Crispy ice cream, giving your answer correct to 2 decimal places.   [1]

(c)   Mr Ho is considering changing the biscuit cone to another type, as shown in
       Figure 4, which costs 5 cents cheaper.
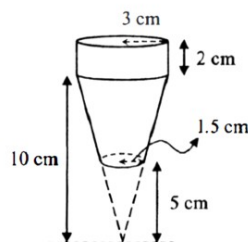
       In Figure 5, the new biscuit cone can be modelled by removing a cone of
       radius 1.5 cm and height 5 cm from another cone with radius 3 cm. In
       addition, a cylinder of height 2 cm and radius 3 cm is mounted on top of the
       cone with the same radius.

Figure 4   Figure 5

As per his usual practice, Mr Ho will fill the new cone with soft ice cream completely before topping up with an amount of ice cream to form a hemisphere at the top as illustrated in Figure 6 below.

Figure 6

Mr Ho thinks that using the new biscuit cone can help to lower the cost price and hence increase his profit.

Would you advise Mr Ho to change to the new biscuit cone?
Support your decision with clear working and reasoning. [5]

2 A sculpture was designed with a conical body and a spherical top. The conical body had a base radius of 0.4 m and a slant height of 1.2 m. The spherical top has a radius of 0.3 m.
   (a)   Find the height of the sculpture. [2]
   (b)   Find the total surface area of the sculpture. [3]
   (c)   Find the volume of the sculpture. [3]
   (d)   The sculpture can be made from either copper, bronze or stainless steel. This sculpture will be displayed on a stand which can support a mass of not more than 2400 kg. Explain which metal should be used to make this sculpture, and support your answer with calculations. [4]

Material	Copper	Bronze	Stainless steel
Density (kg/m^3)	8940	7950	7700
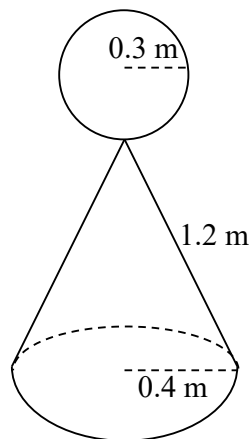
3   Jane attended a pottery workshop and made a ceramic cup consisting of a hollow frustum and a hollow hemisphere as shown below. She was then given a cup holder ring to allow the cup to stand.
The frustum is formed by cutting off the bottom $2x$ cm from an inverted cone of radius 5.6 cm and height $(2x + 4.5)$ cm of the cone.

(a)   Using Figure 1, show that $x = 3.35$ cm.   [2]

(b)   (i)   Assuming negligible thickness of the ceramic cup, show that the total volume of liquid the ceramic cup can hold is 367.81 cm³.   [3]

(ii)   The cup can only be filled up to 85% of its total volume to avoid spillage. Jane decided to pour water into the cup to a height level of 7.35 cm. The radius of the frustum is 5.35 cm when the cup is 7.35 cm high.

(a)   Show that the volume of the liquid in the frustum only is 242 cm³.   [3]

(b)   Hence or otherwise, justify with mathematical workings and a brief explanation on whether Jane is able to avoid spillage.   [2]

5.6 cm

4.5 cm

x cm

2x cm
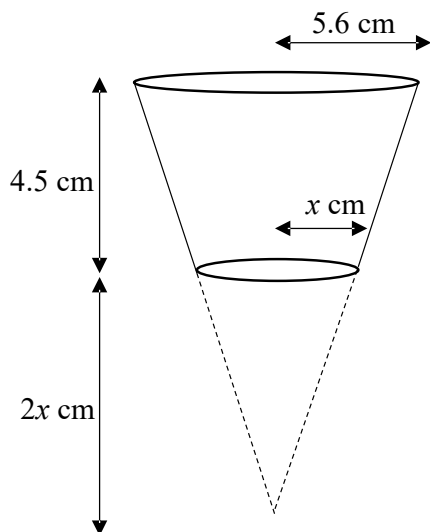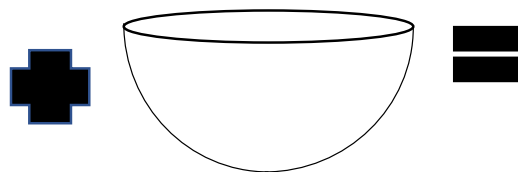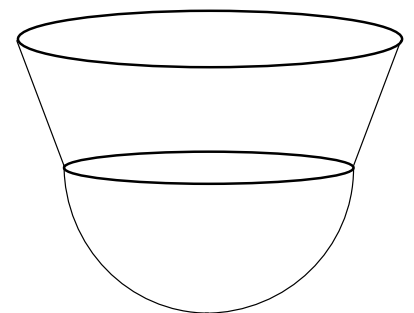
Figure 1

Figure 2

Figure 3

Data Analysis

Chapter Summary

1   Dot diagram

     Advantages : (i)   It is useful for highlighting clusters of a data set.
                 (ii)   It shows the maximum and minimum values of a data set.
                  (iii)   It retains the original data values of a data set.

     Disadvantages : (i)   It is not good for large data sets consisting of more than 30 data values.
                         Too many dots can cause the diagram to appear cluttered.
                   (ii)   It is not good for data sets whose range is too large.
                         This is because we cannot observe useful patterns from the diagram.

2   Histogram

     Advantages: (i)   It can be used for large data sets.
                 (ii)   We can observe the shape of the distribution of data from the bars.
                 (iii)   We can read the frequency of a class from the vertical axis.

     Disadvantages : (i)   We cannot retrieve the original data values from a histogram.
                   (ii)   We cannot compare two data sets in one histogram.

3   Stem and leaf diagrams

     Advantages : (i)   We can observe the shape of the distribution of data from the diagram.
                  (ii)   It shows the minimum and maximum values of a data set.
                  (iii)   It shows the order of the data.
                  (iv)   We can retrieve the original data values from a stem and leaf diagram.
                  (v)   We can use a back to back stem and leaf diagram to compare the distributions of two data sets.

     Disadvantages : (i)   It is not good for large data sets.
                   (ii)   It is not good for a data set with too large or too small a range since it may be difficult to define appropriate stems.

4   Mean

     Ungrouped data : $\dfrac{\text{sum of } x_i}{N}$,

     Grouped data : $\dfrac{\text{sum of } f_i x_i}{\text{sum of } f_i}$

     It is affected by extreme values. Hence, in some cases, it may give a misleading average value.

5   Median

     Arrange N items in a data set in ascending order.
     If N is odd, the median = the middle term.
     If N is even, the median = the average of the middle two terms.

6   Mode

The mode of a data set is the value that occurs most often.
If there are two values that have the highest frequency, this distribution is called bimodal.
If each value in a data set has the same frequency, there is no mode.

Data Analysis

Exam Questions:

Stem and Leaf diagram

1   12 students from Class 3A and Class 3B had their 2.4 km run.
The timings, in minutes, are shown in the stem-and-leaf diagram.

Class 3A		Class 3B
9 8	0	7 8 9
3 3 2 0 1	1	1 1 1 2 2
4 3 3 2 2	2	0 2 4
	3	0

Key (Class 3A)
3|2 means 23 minutes

Key (Class 3B)
2|4 means 24 minutes
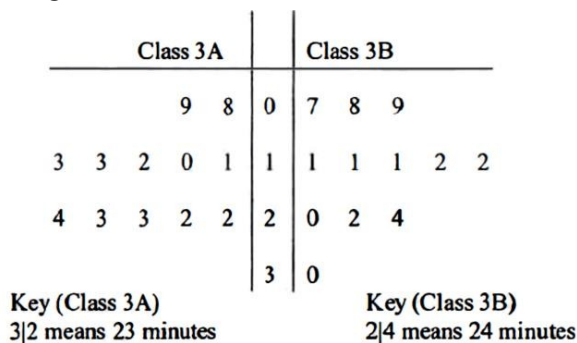
    (a)   Write down the modal timing of students from Class 3B.   [1]
    (b)   Write down the median timing of students from Class 3B.   [1]
    (c)   Write down the median timing of students from Class 3A.   [1]
    (d)   Explain briefly whether students from Class 3A or Class 3B
          performed better.   [1]

2   A group of students took a Mathematics test which has a maximum of 100 marks. The marks are shown in the stem-and-leaf diagram below.

Boys								Girls				
					7	0						
						1						
						2						
						3	7					
				9	6	4						
			3	3	0	5	2	4	4	5		
9	8	6	6	6	4	6	1	2	2	3	5	9
		7	5	4	2	7	0	5				
					8	8	1	5	8			
					8	9	2	7				
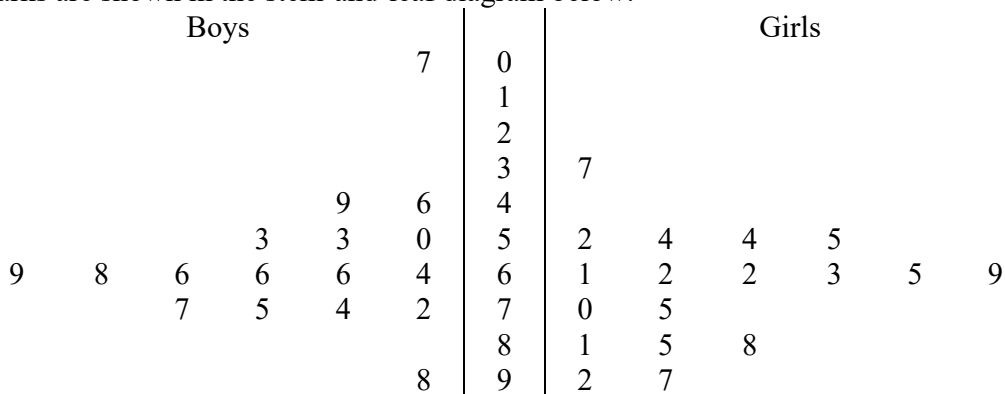
Key: 3|7 means 37 marks

(a)   Write down the total number of students who took the test.
(b)   Write down the modal mark.
(c)   (i)   State the median of the boys marks and the median of the girls marks.
       (ii)   Hence, determine whether the boys or the girls did better for the test.
(d)   Explain why the mean is not an appropriate measure of central tendency for the above data.
(e)   If 20% of the students scored a distinction for the test, write down the minimum distinction mark.
(f)   A student is selected at random. Find the probability that the students mark is less than 50 marks.

3   The following data shows the timing, in seconds, of 20 girls swimming in a 100 m freestyle competition.

Stem	Leaf
6	0 2 2 3 7
7	1 5 6 6 6 8 8 9 9
8	0 0 1 4 4 9

(a)   (i)   Using the information from the stem and leaf diagram, calculate the mean.
       (ii)   What is missing in the stem and leaf diagram shown above?
(b)   (i)   Construct a histogram for the data in the stem and leaf using the following class intervals :
              $60 \leq x < 70$, $70 \leq x < 80$, $80 \leq x < 90$.
       (ii)   By using the information from the histogram, calculate an estimate of the mean of the data.
(c)   Compare and explain briefly the mean calculated in part (a) and (b).

4   Ten students sat for a Mathematics test. Their marks were recorded in the table as shown below.

93	98	81	72	39
45	70	77	85	65

Represent the data on a stem-and-leaf diagram.   [2]

The stem and leaf diagram below shows the scores of 21 students in a Mathematics class test. The full Mathematics mark of the test is 60 marks.

Stem	Leaf
2	5 7 9
3	0 1 y 2 3 9
4	2 4 5 6 6 x 9
5	0 1 3 6 6

Key: 2|5 represents 25 marks

(a)   Given that the modal score is 46 marks and the mean score is 41 marks, find the values of x and y.   [3]

(b)   Find the median score.   [1]

(c)   It was discovered that one of the questions had error and thus all the students were given 1 more mark. How will this affect the mean and median mark? [2]

Dot diagram

1   The dot diagram shows the late-coming records of 15 students in a certain class for Term 3.
    (a)   Find the mean number of times the students were late.
    (b)   Find the percentage of students that were late for more than 3 times.
    (c)   Which measure is the most appropriate as a measure of central tendency in this case? Explain your answer.
    (d)   Another student joined the class. He was late for x number of times in Term 3. Express the new mean of the 16 students in terms of x.

Number of times students were late
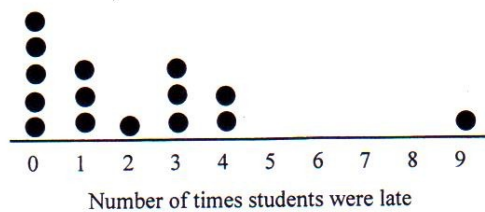
2   In a chicken farm, a study was conducted on the growth of 20 chicks fed with a special feed. The increase in the mass of each chick after a week was measured and presented in a dot diagram below.

Increase in mass (g)

34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50
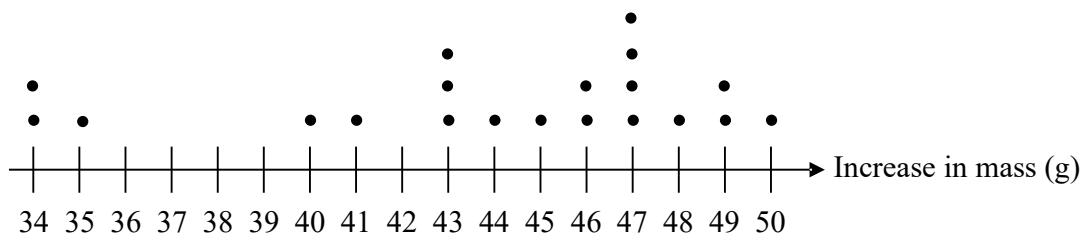
    (a)   Find the percentage of chickens which had grown at least 43 grams after 1 week.   [2]
    (b)   Find the median and modal increase in the mass after one week.   [2]
    (c)   Given that mean increase in mass is 43.9 g, explain if the mean, mode or median, is the most suitable measure of the effectiveness of the special feed.   [2]

3   The dot diagram shows the CIP hours obtained by Sec 5/2 of Coral Secondary School for the year 2015.
   (a)   Write down the number of students in Sec 5/2.
   (b)   Find the mean number of CIP hours.
   (c)   Due to a calculation error, the CIP hours of all students who have done CIP for at least 3 hours has been over-reported by 1 hour each. Find the actual modal number of CIP hour(s).

Number of CIP hours
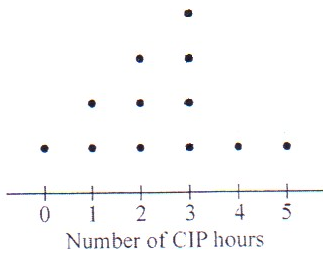
Frequency table

1 The following table shows the distances in kilometres of 30 trips made by a taxi.

Distance (km)	$0 < x \le 10$	$10 < x \le 20$	$20 < x \le 30$	$30 < x \le 40$
Frequency	6	12	7	k

    (a)   Find the value of k.   [1]

    (b)   State the median range of distance made by the taxi.   [1]

    (c)   Calculate an estimate of the mean distance made by the taxi for the 30 trips.[2]

2 The table below shows the amount of pocket money a class of thirty students receive each week.

Amount of pocket money ($)	15	16	17	18	19	20
Number of pupils	2	3	x	9	y	4

    (a)   Given that the mode is $18, find the largest possible value of x.
    (b)   With x taking the largest value from part (a),
        (i)   find the value of y,
        (ii)   hence, calculate the mean amount of pocket money the students receive each week.

3 (a) Lisa is thinking of five numbers, a, b, c, d and e in ascending order.
The mean of these numbers is 13, the median is 14 and the mode is 18.
The smallest number is one-sixth of the largest number.
Find the values of a, b, c, d and e.   [5]

    (b)   The table below shows the number of days that a class of students were late in a week.

Number of days late	0	1	2	3	4	5
Number of students	22	8	x	5	0	1

        (i)   If the mean number of days late is 0.9, find the value of x.   [3]
        Students who are late for 3 or more days in a week will have to serve detention.
        (ii)   What fraction of the class has to serve detention?   [1]
        (iii)   The data of one student who was absent the whole week was not included. If this data is included, will the median of the distribution be affected? Explain your answer.   [2]

4   (a)   The table below shows the History marks of all the Secondary One students.

Marks (x)	Frequency (f)
$0 < x \leq 20$	45
$20 < x \leq 40$	p
$40 < x \leq 60$	103
$60 < x \leq 80$	95
$80 < x \leq 100$	35

        (i)   If the modal class is $40 < x \leq 60$, find the largest possible value
              of p.   [1]

        (ii)   If the median class is $20 < x \leq 40$, find the smallest possible value
              of p.   [2]

    (b)   The table below summarises the time taken, x minutes, by 100 female
            competitors in a 5 km race. Calculate an estimate for the mean time taken by
            the competitors.   [2]

Time (x minutes)	Frequency (f)	Mid-value (x)	fx
$15 < x \leq 20$	7		
$20 < x \leq 25$	35		
$25 < x \leq 30$	27		
$30 < x \leq 35$	29		
$35 < x \leq 40$	2		

Misc

1   Four numbers have a mean of 34, a mode of 30 and a median of 31. Given that all numbers are integers, find the value of the largest number.   [3]

Probability

Chapter Summary

1   A random experiment is a process whose result cannot be predicted with certainty.

2   An outcome is a possible result of a random experiment.

3   The sample space is the list of all possible outcomes of a random experiment.

4   An event is a collection of outcomes from the sample space that we are considering in particular.

5   Probability of an event E, $P(E) = \dfrac{n(E)}{n(S)}$, where n(E) is the number of outcomes in the event E

                      n(S) is the number of outcomes in the sample space S.

6    $0 \leq P(E) \leq 1$
     0 signifies that the event will SURELY happen.
     1 signifies that the event will NEVER happen.

7   P(not E) = 1 – P(E)

Probability

Exam Questions:

1   (a)   A two-digit number is formed at random using the digits 1, 4 and 5 without repeated digits.
(i)   Write down the sample space.
(ii)   Find the probability that the 2 digit number formed is an even number.
(b)   An unbiased eight-sided die with its faces labelled with numbers 1 to 8 is rolled and the number on its top face is noted. Find the probability that the number shown is
(i)   a prime number, and
(ii)   a square number.

2   A two-digit number is formed at random in a single event using the digits 2., 5 and 7 with repetition of digits allowed.
(a)   List all possible two-digit numbers.
(b)   Find the probability of
(i)   forming an even number,
(ii)   forming a number x, such that $52 < x < 77$,
(iii)   not forming a prime number.

3   A bag contains 10 red marbles, 5 blue marbles and 3 yellow marbles. A marble is drawn at random from the bag.
(i)   Find the probability that the marble drawn is red.   [1]
(ii)   How many more blue marbles must be placed in the bag so that the probability of drawing a blue marble would be $\frac{3}{4}$?   [2]

4   The pie chart shows the number of different coloured marbles in a box.
     (a)   A marble is drawn at random. Find the probability that the marble is
            (i)   red,   [1]
            (ii)   not green.   [1]
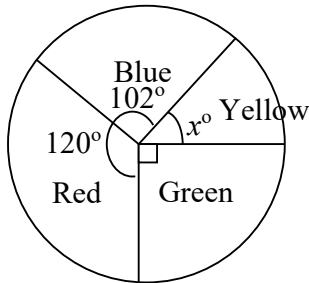
     (b)   If the probability of drawing a yellow marble is $\dfrac{2}{15}$, find the value of x.   [2]

     (c)   If there are 170 blue marbles, find the total number of marbles in the box.   [1]

5   (a)   Each of the numbers 2, 3 and 6 are written on a card. Two cards are drawn at
            random to form a two-digit number. Find the probability that the number
            formed
            (i)   is a prime number,   [2]
            (ii)   is a multiple of 5.   [1]

     (b)   UNO is an American card game played with a specially printed deck of cards.
            The deck consists of 108 cards (76 normal cards, 24 action cards and
            8 wild cards) in five colours (red, green, blue, yellow and black).

            The colour and distribution of a deck is shown below.

Colour	Red	Green	Blue	Yellow	Black	Total
Normal cards labelled 1 to 9	18	18	18	18		72
Normal cards labelled 0	1	1	1	1		4
Action cards labelled "skip, "draw two and "reverse	"6	6	6	6		24
Wild cards					8	8

            (i)   Find the probability of picking a blue normal card from the deck.   [1]
            (ii)   Winston removed x black cards from the deck and the probability of
                   black cards was reduced to —. Form an equation in x and solve for the
                   number of black cards Winston removed.   [2]

Quadratic Graphs

Chapter Summary

1. $y = ax^2 + bx + c$, where $a \neq 0$.
   (i) It is in the shape of a parabola.
   (ii) It opens upward and has a minimum point when $a > 0$.
        It opens downward and has a maximum point when $a < 0$.
   (iii) It has a vertical line of symmetry through its minimum or maximum point.
   (iv) It cuts the x-axis at 0, 1 or 2 points.
   (v) It cuts the y-axis at only 1 point.

Quadratic Graphs

Exam Questions:

Plotting

1   The variables x and y are connected by the equation $y = 2x^2 - 5x + 7$.
Some values of x and their corresponding values of y are given in the table below.

x	4	3	2	1	0	1	2
y	5	p	9	10	7	0	11

   (i)   Find the value of p.   [1]
   (ii)   Draw the graph of $y = 2x^2 - 5x + 7$ for $4 \le x \le 2$ on a graph paper.   [2]
   (iii) From your graph, state the equation of the line of symmetry.   [1]
   (iv)   By drawing a suitable line, solve the equation $2x^2 + 5x \quad 4.5 = 0$.   [2]

2   Answer the whole of this question on a sheet of graph paper.

   A stone was thrown from a bridge.
   Its position during the flight is represented by the equation $y = 2x^2 + 3x + 20$, where
   y metres is the height of the stone above the water and x is the time in seconds after
   the stone is thrown.

   Some corresponding values of x and y are given in the following table.

x	0	0.5	1	2	3	4	5
y	p	21	21	18	q	0	15

   (a)   Find the values of p and q.   [2]

   (b)   Explain the significance of the value of p.   [1]

   (c)   Using a scale of 2 cm to represent 1 second, draw a horizontal x-axis for
         $0 \le x \le 5$.
         Using a scale of 2 cm to represent 5 metres, draw a vertical y-axis for
         $15 \le y \le 25$.
         On your axes, plot the points given in the table and join them with
         a smooth curve.   [3]

   (d)   Use your graph to find the greatest height reached by the stone.   [1]

   (e)   Find the length of time for the stone to have a height greater than
         20.5 metres.   [2]

   (f)   How long does it take for the stone to hit the water?   [1]

3   Answer the whole of this question on a sheet of graph paper.
The variables x and y are connected by the equation $y = 5 + 1.5x - 0.5x^2$.
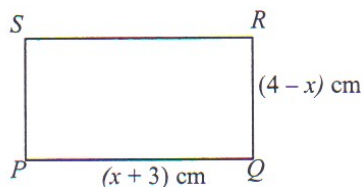Some corresponding values of x and y are given in the table below.

x	3	2	1	0	1	2	3	4	5	6
y	4	0	3	5	6	p	5	3	q	4

(a)   Find the value of p and of q.   [2]

(b)   Using a scale of 2 cm to represent 1 unit, draw the horizontal x-axis for
$3 \le x \le 6$.
Using a scale of 2 cm to represent 1 unit, draw the vertical y-axis for
$4 \le y \le 7$.
On your axes, plot the points given in the table and join them with
a smooth curve.   [3]

(c)   Use your graph to find
(i)   the maximum value of y,   [1]
(ii)   the value of y when $x = 1.5$ [1]

(d)   By drawing a line on your graph in (b), solve the equation
$4 + 1.5x - 0.5x^2 = 0$.   [2]

4   Answer the whole of this question on a sheet of graph paper.
The diagram shows a rectangle PQRS. PQ = $(x + 3)$ cm and QR = $(4 - x)$ cm.

(a)   Show that the area, A cm^2, of the rectangle, is given by
$A = (x^2 + x + 12)$ cm^2.

(b)   (i)   Some corresponding values of x and A are given in the following table.
Find the value of p and of q.

x	2	1	0	1	2	3	3.5
A	6	p	12	q	10	6	3

(ii)   Using a scale of 2 cm to represent 1 unit on the horizontal x-axis and
1 cm to represent 1 unit on the vertical A-axis, draw the graph of
$A = (x^2 + x + 12)$ for $2 \le x \le 3.5$.

(c)   Hence, using your graph, find
(i)   the values of x when the area of the rectangle is 8 cm^2,
(ii)   the maximum area of the rectangle,
(iii)   the length of PQ when its area is maximum.

5   Answer the whole of this question on a sheet of graph paper.
The total monthly cost of producing toys consists of fixed factory rental cost and variable production cost.
The net profit y (thousand) dollars, of a company when x (hundred) toys were produced is given by the equation $y = x^2 + 6x - 3$.
The corresponding values of x and y are given in the table below.

x (hundred)	0	1	2	3	4	5	6
y (thousand)	3	2	5	6	5	2	3

(a)   Using a scale of 2 cm to represent 1 hundred toys on the horizontal x-axis and a scale of 2 cm to represent 1 thousand dollars on the vertical y-axis, draw the graph of $y = x^2 + 6x - 3$ for $0 \leq x \leq 6$.

(b)   Using your graph, estimate
     (i)   the profit earned when 150 toys are made,
     (ii)   the number of toys to produce in order to obtain maximum profit,
     (iii)   the monthly fixed factory rental when no toys are produced,
     (iv)   the least number of toys the company must produce in order not to suffer a loss.

Theory

1   The diagram shows the graph of $y = (3 - x)(x + 4)$.
   (a)   Find the coordinates of A and B.
   (b)   Find coordinates of C.
   (c)   Write down line of symmetry.
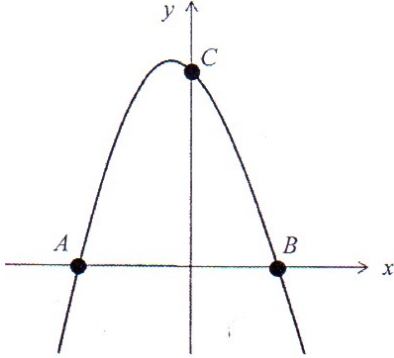
2   The graph of $y = 3x^2 - 2x - 8$ is shown in the diagram below. It intersects the y-axis at P and the x-axis at Q and R. Find the coordinates of P, Q and R.   [4]